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PUTTING THE MATERIAL IN THE RIGHT PLACE

Investigations into the sustainable use of structural materials to reduce the embodied environmental impact of building structures

MPhil thesis by Clement THIRION

carried out at University College London

in partnership with Expedition Engineering Ltd

ABSTRACT

The contribution of the built environment to current environmental issues is significant. The present thesis investigates the potential to reduce the initial embodied environmental impact of building structures through a more sustainable use of materials. Following a review of the growing importance of embodied environmental aspects in the whole life environmental impact of buildings, a framework is proposed for investigation into this topic. Two case-studies follow which relate to two areas of the framework.

The first study focuses on the potential to reduce the quantities of materials required in the most common steel structural element, steel I-beams, by varying their section along their length. It has been instigated by the work done by a rolling mill manufacturer who is developing a method to roll varying section beams. For a configuration largely representative of contemporary office buildings, significant savings are shown to be possible. These are discussed in light of their implication on the complexity of the required fabrication method and the constraints associated with the building typology considered.

Floors are the structural element concentrating the largest proportion of carbon embodied in building structures. The second case-study looks into the impact of two design choices on the carbon intensity of a commonly used flooring system: flat slabs. Significant savings are shown to be achievable which do not require technological advances but a greater attention to design choices. A number of such studies focused on a range of structural elements will develop our understanding of how to design concrete structures to minimise their embodied carbon content, and enable designers to make informed design decisions. It would also constitute a useful framework for a review of current design practice aimed at quantifying the savings obtainable from a more environmentally conscious practice, making best use of currently commonly available design and construction techniques.

Keywords: building structure, initial embodied environmental impact, embodied carbon, material-efficiency, steel I-beam, shape optimisation, fabrication, flat slab

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INTRODUCTION

Throughout history, architecture has constantly been shaped by its context, or in other words technical, social and economic aspects. As these evolve, the architecture produced changes and adapts. Today, environmental issues are becoming so important that they can no longer be ignored. And the built environment, which is known for having a significant impact on the environment, needs to change to reflect this. Indeed, for certain categories of impacts the built environment is one of the largest contributors. Taking the example of one component of this influence, its contribution to climate change, the built environment is responsible for around 40% of worldwide greenhouse gas emissions.

The environmental impact of a building over its life span, its whole life environmental impact, has two origins: the *operational impact* arises from the energy used to operate the building, to provide heating, cooling and lighting, and to power the appliances in the building. The second part is called the *embodied environmental impact*. It is the impact that is associated with the 'making' of a building, through its initial construction, its maintenance over its life time and finally its demolition and disposal at the end of its life.
If building regulations in most countries, and certainly in the UK, have started focusing on operational aspects several decades ago, embodied aspects are currently left largely unaddressed. However, due to improvements in the operational energy-efficiency of buildings, the relative significance of their embodied environmental impact is growing. It is this part of the whole life impact of buildings which this thesis covers. More particularly, it looks into the impact associated with one key component of buildings: their structure.

Two main areas of investigation can be highlighted: the first one relates to the recurring and demolition components of their embodied environmental impact. Termed *life time structural engineering*, this range of measures focuses on the life of structural components and materials following the construction of the building. The second area of investigation is called *eco-efficiency* and covers all measures aiming at reducing the initial embodied environmental impact of structures, that associated with their construction.

Through two case studies, this thesis investigates one particular aspect of the eco-efficiency of structures: the *potential for reducing the initial embodied environmental impact of building structures through the sustainable use of given structural materials*, or put more simply '*Putting the material in the right place*'. Measures considered include those resulting in reductions in the quantities of material required for a given purpose, but also measures resulting in a reduction in embodied environmental impact through a particular use of materials as will be further explained.

This thesis is organised in five main chapters.

Chapter 1 aims at setting the scene and explaining how the range of measures considered in this thesis relates to the overall range of possible measures. Building on previous work by the author, it results in the production of a framework for the investigation of the range of measures covered under the topic of *reducing the initial embodied environmental impact of building structures through the sustainable use of structural materials*. It is followed by the presentation of two case studies.

The first case study, presented in *Chapters 2 and 3*, is dedicated to the study of the shape optimisation of steel I-beams. It has been instigated by the work done by a leading rolling mill manufacturer who is developing a method to roll bespoke steel I-beams with a section which varies along the length of the element. Such beams have the potential to result in significant material savings, and consequently in direct reductions in embodied environmental impact. This study aims at informing the development of this fabrication method by providing an understanding of the geometry of the beams which such a rolling mill should target, balancing the benefits in weight savings to be obtained from an increased complexity in the geometry of the beams with the corresponding impact on the required rolling mill. *Chapter 2* presents the methodology and theory used in the investigation of this question. The results obtained are then presented, interpreted and discussed in *Chapter 3*.

The second case study, presented in *Chapters 4 and 5*, presents an investigation into the design of reinforced concrete flat slabs to minimise their embodied environmental impact. Flooring systems concentrate a large proportion of this impact, and designing them with a view to reduce it may hence result in significant savings. This study aims both at providing design guidelines for designers, and contributing to the understanding of how to design structures with low embodied environmental impact, which may be used for a review of current structural design practice as will be explained. *Chapter 4* presents the scope, methodology and theory adopted for the investigation. The results are then presented and discussed in *Chapter 5*.

Chapter 6 concludes this thesis by summarising the results obtained through the case studies and discussing possible areas for future work.

Chapter 1
Background

This chapter introduces the context in which the investigations presented in this thesis take place. It develops previous work by the author (Thirion 2010, 2010b).

The role of the built environment in current environmental issues is first reviewed. This is followed by a discussion about the growing relative significance of the environmental impacts linked to the '*making*' of buildings, as opposed to their operation. A possible definition of the whole life embodied environmental impact of buildings is then given which highlights the challenges and uncertainties associated with the calculation of such an impact, as well as existing opportunities to reduce it. These opportunities are then presented in greater detail, and the particular range of possible measures covered under the title of this thesis '*Putting the material in the right place: reducing the initial embodied environmental impact of building structures through the sustainable use of structural materials'* are presented and a classification is proposed.

1.1 THE BUILT ENVIRONMENT: ASSOCIATED ENVIRONMENTAL ISSUES

Environmental issues have gained significant consideration over the past five decades: if in the 1960s only a circle of experts were conscious of the finiteness of the earth's resources and of the fragility of its cycles, awareness of these issues has recently become more global as the extent of their potential consequences has appeared more clearly.

1.1.1 Current environmental issues

The global environmental issue currently perceived as the most important by the world community is climate change. Strong evidence suggests that the recent global temperature increase observed at the surface of the Earth is linked to human activities through the emission of additional greenhouse gases (GHGs) in the atmosphere. The rise in temperature predicted by the Intergovernmental Panel on Climate Change (IPCC 2001), if GHGs emissions do not peak and start decreasing soon, is such that it would have extreme consequences on humans and their natural environment.

Climate change is however not the only environmental issue of concern currently: fossil fuel depletion (BP 2010), excessive mineral resource extraction and disproportionate creation of waste (Davis Langdon 2009) are as many topics which need to be addressed.

1.1.2 Impact of the built environment

In a previous review, Thirion (2010) pointed out the significant role which the built environment plays in each of these issues.

The United Nation Environment Program (UNEP 2007) reports that buildings consume between 30% and 40% of all the primary energy used worldwide. In the UK, this proportion is even higher as 45% of the country's total energy use is utilised in buildings (CIBSE 2006). According to Smith (2005), buildings are also responsible for a similar proportion of the total GHGs emissions which makes the built environment the largest contributor to GHGs emissions both in the UK and worldwide.

Each year approximately 3 billion tonnes of raw materials are used in the manufacturing of building products and components worldwide. The fact that this represents 40% to 50% of the total flow in the global economy (UNEP 2007) gives a sense of magnitude of the impact of the sector. In the UK, the construction industry uses over 400 million tonnes of materials, which makes it the largest consumer of natural resources in the country (Davis Langdon 2009). The production and use of these building materials account for around 10% of all UK energy use (Anderson 2009) and a similar proportion of the country's carbon emissions.

Furthermore, UNEP (2007) state that between 30% and 40% of all solid waste generated worldwide is produced by the construction sector. Results are similar in the UK, where the construction industry is responsible for 120 million tonnes of construction, demolition and excavation waste every year. This represents a third of all waste created in the UK (Davis Langdon 2009).

The above examples demonstrate that the built environment plays a significant role in current environmental issues. But, in most countries, regulations currently only account for part of it. This is particularly true of GHGs emissions, as discussed in the following section.

1.2 IMPORTANCE OF THE 'MAKING' OF BUILDINGS: INTRODUCING EMBODIED CARBON

The environmental impact of a building over its lifespan, its whole life environmental impact, has two origins:

- The *operational impact* arises from the energy used to operate the building.
- The *embodied impact* is that associated with the 'making' of the building.

In this section it is argued, through the particular case of carbon emissions, that the embodied component of the whole life environmental impact of new buildings is significant and needs to be addressed.

Global Warming Potential (GWP) is a relative measure of how much a particular greenhouse gas contributes to global warming when in the atmosphere. To define GWP, carbon dioxide is used as a reference: the quantities of GHGs associated with a particular phase of the life-cycle of a product or building are hence given in carbon equivalent. For this reason, in the remaining of the text, '*carbon emissions*' should be understood as carrying the same meaning as '*GHGs emissions*'.

1.2.1 Whole life carbon emissions

The two origins highlighted above for environmental impact also apply to carbon emissions:

- *Whole life operational emissions* arise from the energy used to operate the building; in other words to provide heating, cooling and lighting and to power the appliances in the building.
- The remaining part is called *whole life embodied carbon emissions* and can be considered as made up of three parts (Yohanis and Norton 2002):
    - *Initial embodied carbon* is the carbon emitted to produce the building initially, accounting for the extraction, processing and manufacture of the materials and building components, as well as their transportation and assembly on site. This component has two origins:
        - The required consumption of fossil fuel; this part of the initial embodied carbon is directly proportional to the amount of energy consumed (initial embodied energy).
        - The chemical reactions involved in the manufacture of the materials; the most obvious example being the carbon dioxide created when converting limestone into cement.
    - *Recurring embodied carbon* corresponds to the maintenance and refurbishment of the building over its lifetime.
    - *Demolition carbon* is that associated with the demolition and disposal of the building at the end of its life.

This split is summarised on Figure 1. Figure 2 gives an indicative profile of the operational and embodied carbon emissions in the life-cycle of a building.

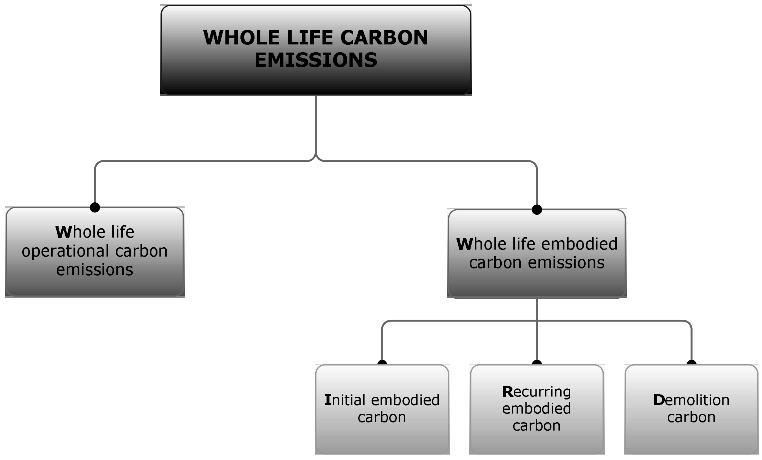

Figure 1: Components of whole life carbon emissions

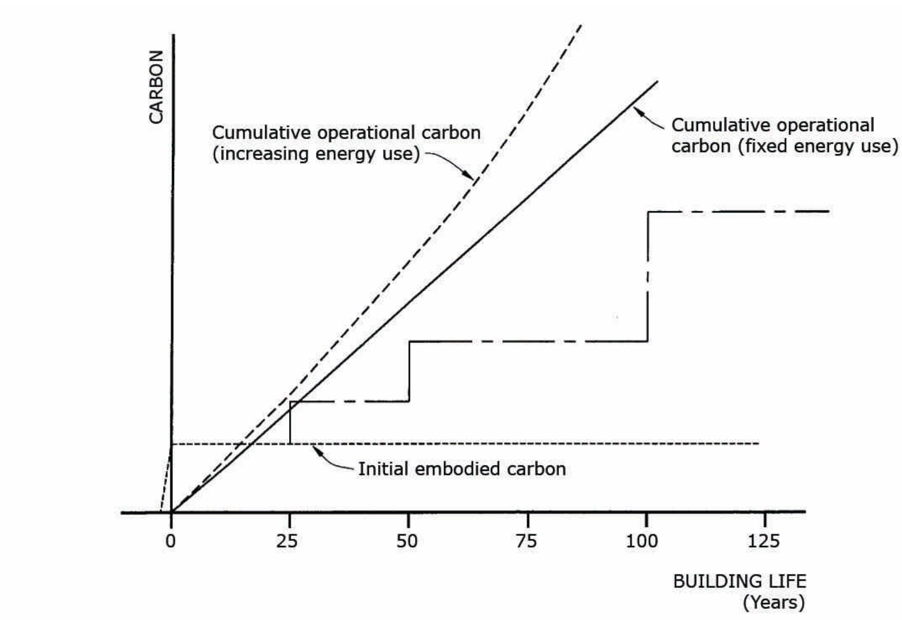

Figure 2: Operational and embodied carbon along a building's life-cycle (adapted from Yohanis 2002)

1.2.2 Operational and embodied carbon: an evolving split

In many countries, including the UK, building regulations only aim at regulating operational emissions. There are several reasons for this:

- In the UK, building regulations were introduced in 1965 to guarantee building occupants with a minimum level of comfort and hence focused on operational aspects.
- The two oil crises of the 1970s resulted in the introduction of regulations to reduce the energy consumption of buildings. A split of 90% to 10% between operational and embodied energy is understood as representative of new buildings at that time (Sartori 2007). Moreover, being given the slow rate of replacement of the building stock (Roberts 2008), the regulations rightly addressed operational aspects.
- The past decades have seen a shift in focus from energy to carbon emissions as global warming grew into a greater concern. Again, due to the slow replacement rate of the building stock, and to the fact that operational carbon emissions still represented a significant proportion of the whole life emissions of new buildings, the focus has remained on operational aspects.

However, as demonstrated in a previous review by the author (Thirion 2010), as the building regulations related to operational aspects are becoming more stringent, embodied carbon grows to represent a greater proportion of the whole life carbon emissions of new buildings.

The following review by the Building Research Establishment (BRE) of the evolution of the yearly carbon emissions associated with the operation of two building types gives a meaningful summary of the evolution of operational carbon emissions over the past decades:

- Figure 3 presents the evolution of operational carbon emissions produced yearly by a typical new semi-detached house between 1981, the year when the building regulations associated with the conservation of fuel and power were first introduced, and 2006. This shows a reduction of as much as 47% in yearly operational carbon emissions.
- Figure 4 presents the same evolution for a typical new air-conditioned office. It shows a similar reduction of 45% in yearly operational carbon emissions between 1995 and 2006.

Data regarding the relative proportions of operating and embodied carbon in the life of a building is relatively rare for recent energy-efficient buildings. Thormark (2002) found that the initial embodied energy in a low energy consumption house in Sweden represented more than 40% of the whole life energy requirement over a 50 year assumed lifespan. In the UK, Rawlinson (2007) found a similar result for complex commercial buildings.

Although only a few results are available on this subject, they are in agreement with the reductions in operational carbon highlighted above. The growing relative importance of embodied carbon is likely to continue in the coming years as new regulations are introduced which push the operational energy-efficiency of buildings further towards '*zero-carbon*' buildings (NHBC 2009) – understand zero *operational* carbon. Figure 5 summarises the past and projected evolution of the split between operating and embodied carbon in new buildings and highlights that as buildings tend towards zero-carbon buildings, embodied carbon tends to represent the totality of the whole life carbon emissions of the building.

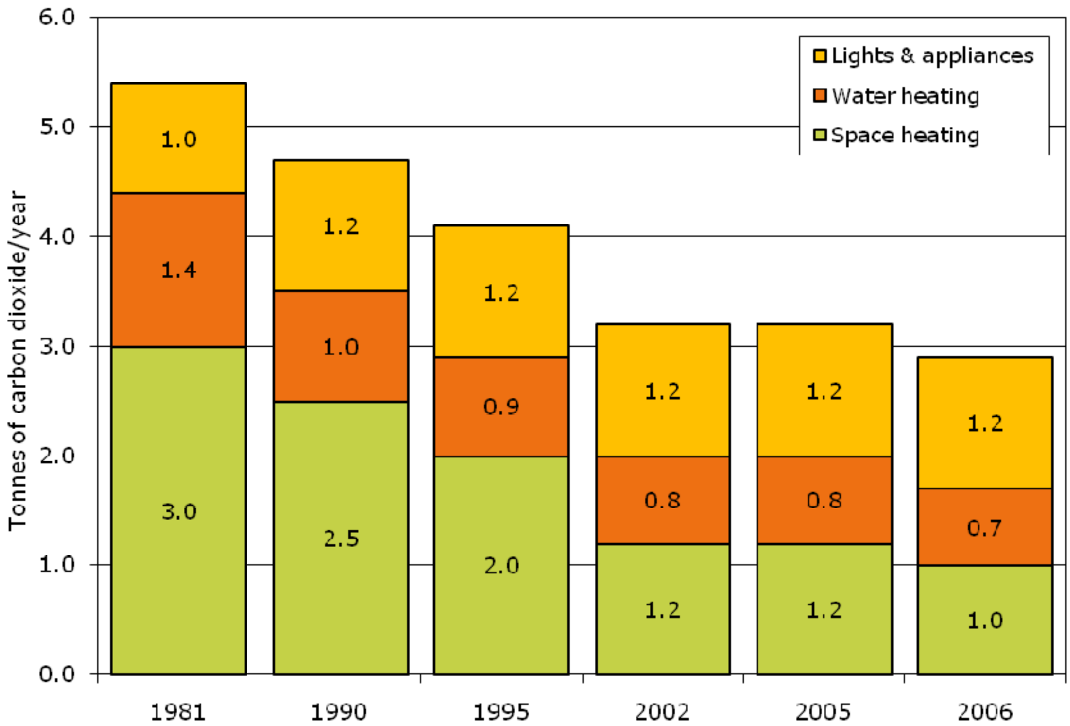

Figure 3: Evolution of operational carbon dioxide emissions from a typical new semi-detached house complying with successive versions of Part L of the UK building regulations (BRE 2006)

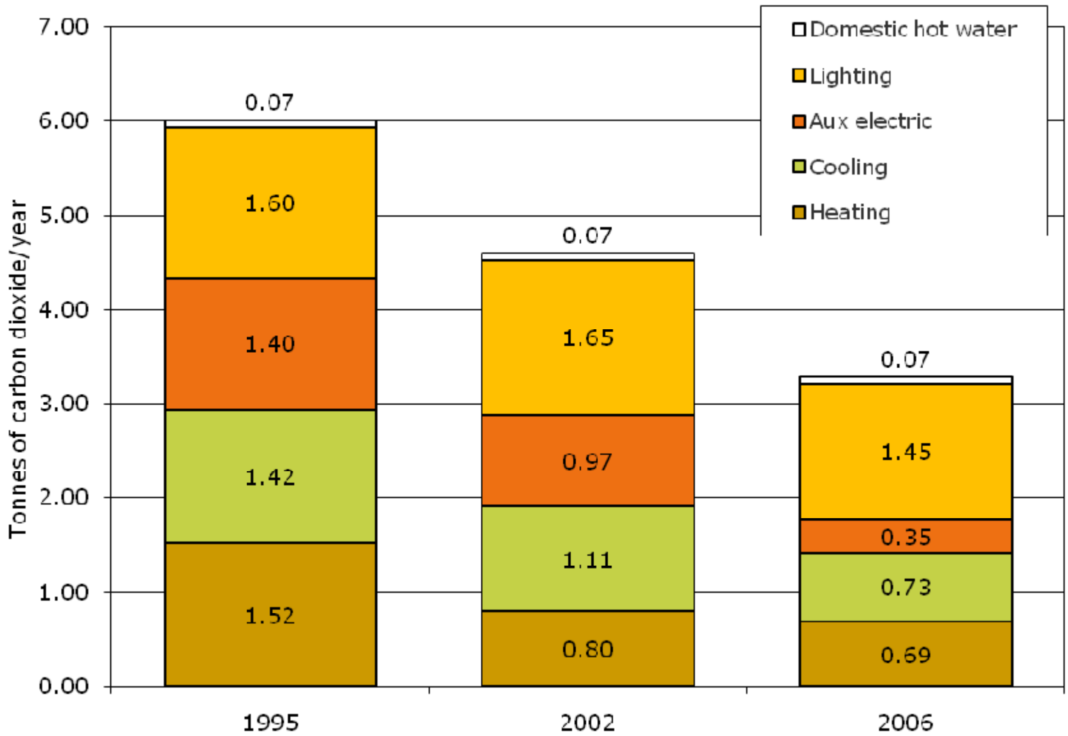

Figure 4: Evolution of operational carbon dioxide emissions from a new air-conditioned office complying with successive versions of Part L of the UK building regulations (BRE 2006)

1.2.3 Additional reasons to account for embodied carbon/energy

The growing relative importance of embodied carbon in the whole life-cycle of new buildings points out the importance of accounting for embodied aspects. As previously indicated by the author (Thirion 2010), additional reasons exist why these should be taken into account.

First there is a concern that savings in operational carbon might require the use of high embodied energy components and, as explained by Thormark (2006), there is a risk that "*the total energy needed in a low energy building may be even higher than in a building with a higher amount of energy needed for operation*". For this reason, a proper whole life-cycle approach to the issue of the environmental impact of new buildings is required.

Additionally, accounting for embodied environmental impacts would enable designers to take the most appropriate decisions: as the first measures to increase the energy-efficiency of a building have been implemented, further reductions of the building's carbon footprint associated with operational aspects become harder and more costly. As a consequence, reducing the embodied carbon content of a building might be an easier win in comparison to measures affecting operational aspects. Aiming at minimising whole life carbon, rather than operational carbon, would hence give designers enhanced flexibility.

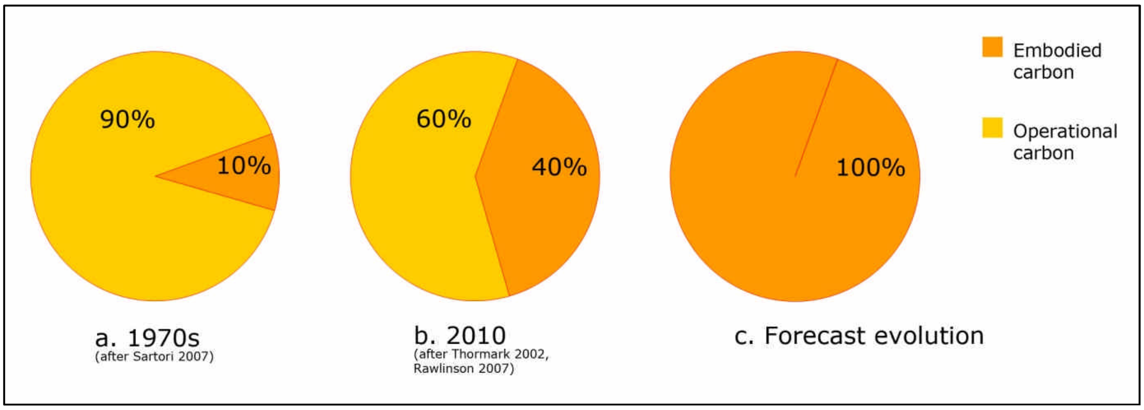

Figure 5: Diagram summarising the projected evolution of the split between operating and embodied carbon in new buildings

1.3 WHOLE LIFE EMBODIED ENVIRONMENTAL IMPACT OF BUILDINGS

The review presented previously highlights that the relative importance of embodied carbon in the whole life emissions of new buildings is growing and justifies the need to start tackling the embodied aspects of the environmental impact of new buildings.

As previously discussed, climate change is only one of the environmental issues to which the built environment contributes. In this section, which expands on previous work by the author (Thirion 2010), a possible definition of the embodied environmental impact of a building, which is also applicable to building components or materials, is reviewed. It is the opportunity to introduce a technique commonly used in the assessment of this impact: Life-cycle assessment (LCA).

Life-cycle assessment techniques have not been directly applied in the work presented in this thesis and the following description does not aim at covering all the technicalities involved. Rather, it is the opportunity to highlight the challenges and uncertainties associated with the exercise. It also constitutes a useful basis for the next section which reviews opportunities to reduce the embodied environmental impact of building structures.

1.3.1 Life-cycle assessment

A life-cycle assessment is a quantitative environmental analysis of a product to measure the environmental impacts associated with each stage of its life-cycle. The diagram in Figure 6 illustrates the life-cycle of a building. The central column of the diagram represents the successive phases in the building's life-cycle, from construction to demolition. The left and right columns indicate respectively the resource input necessary to the completion of each stage, and the resulting emissions. The diagram has been drawn for buildings, but the same principle also applies to single building components or materials.

In the first phase of an LCA, which is called the life-cycle inventory (LCI), the resources necessary to and emissions generated by each stage of the life-cycle are quantified. This is, in effect, a large-scale accountancy exercise which is made extremely complex by the large number of factors on which it depends. Such factors include:

- The energy mix used for the production of each material and component used in the building.
- The proportion of materials which gets recycled and the proportion of elements which get reused when the building is demolished. Figure 6 displays a series of feedback loops indicating components and materials which re-enter the cycle as they get recycled or reused.
- The distance by which each material and component has to be transported to be brought to site.

The presence of so many factors means that producing an LCA for a building, or even a simple building component, can be an extremely complex and time-consuming exercise.

In its purest form, an LCA will account for all stages of the life-cycle of a building, product or material, from the extraction of the raw materials through to its end of life. This approach is the most holistic approach possible and is adequately termed '*cradle-to-grave*'.

Alternative boundaries can however be selected which only consider part of the life-cycle. The two approaches most commonly used are '*cradle-to-gate*' and '*cradle-to-site*'.

In a '*cradle-to-gate*' approach, all stages are accounted for from the extraction of the materials to the manufacture of the building components, as indicated in Figure 6. The accountancy exercise stops at the gate of the factory, hence the name.

A '*cradle-to-site*' approach covers all the stages considered in a cradle-to-gate approach plus those associated with the transportation of the components to the building site as indicated in Figure 6.

These boundaries are sometimes used as alternatives to a cradle-to-grave approach, either because they cover the most significant phases of the life-cycle of the system studied, or because data for the other stages of the life-cycle is not available or varies too much to be representative.

Recently, a database has been created which attempts to list embodied carbon and embodied energy intensities for a broad range of materials. The *Inventory of Carbon and Energy* (ICE) (Hammond 2011) has been developed at Bath University from an extensive literature review of materials life-cycle assessments. The data gathered by the authors highlights that, for a single material type, significant variations occur which may be due to changes in the factors listed above (energy mix etc.) or to the set of boundaries considered. The authors of the database decided to organise it to provide best estimate values of the intensities along with expected uncertainties. The example of concrete highlights the magnitude of these uncertainties, since for this material they are as much as ±30%.

The database is built on the basis of a cradle-to-gate approach, meaning that any energy consumption or carbon emission associated with what happens once the materials have left the factory is not accounted for, which obviously gives rise to further uncertainties.

Since its publication, the ICE database has met a strong success within the construction industry and with building designers in particular. The existence of such uncertainties, however, highlights the need to use the numbers in the database with great care when making design decisions based on them. This question will be further discussed in one of the two case-studies presented hereafter.

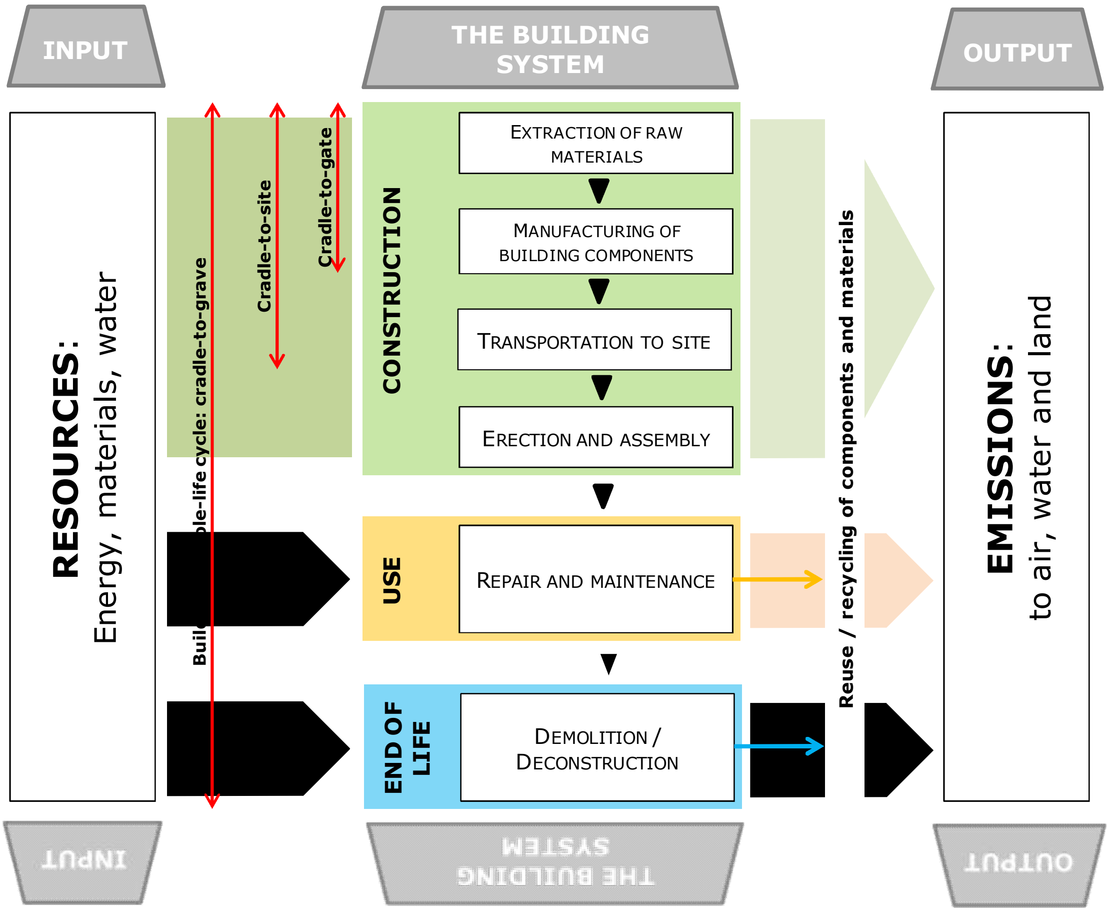

Figure 6: Diagrammatic representation of the life-cycle of a building (after Anderson 2009 and Sartori 2007)

1.3.2 Environmental impacts

Mentioning the LCI above, it was noted that all resource inputs and emissions corresponding to each stage of the life-cycle should be quantified. In fact, only the inputs and emissions considered as having an impact on the environment are quantified. This leads to the question of the definition of environmental impact.

A review of the literature about the embodied environmental impacts of buildings shows a strong focus on carbon dioxide emissions and energy use, which are often the only resources and emissions accounted for in an LCI. This is due in great part to the fact that energy use was the first impact to be considered within the context of the two energy crises of the 1970s and 1980s, and that global warming is currently perceived as the most important environmental issue, leading to a strong emphasis on carbon emissions.

However, as explained previously, the construction, maintenance, operation and disposal of buildings also impact on other aspects. The Building Research Establishment (BRE) have carried out comparative studies of embodied environmental impacts for a series of building products which they published in their guide *Green Guide to Specification* (Anderson 2009). The definition they use for 'embodied environmental impact' consists of 13 different environmental impact categories which, according to the authors, "*reflect the generally accepted areas of concern related to the production of building materials in the UK*". These impacts, as well as the quantities through which they are measured, are listed in Table 1 below.

Environmental impact	Quantity measured
Climate change	Global warming potential or GHG emissions
Water extraction	Water consumption
Mineral resource extraction	Metal ore, mineral and aggregate consumption
Stratospheric ozone depletion	Emissions of gases impacting ozone layer
Human toxicity	Pollutants toxic to humans
Ecotoxicity to freshwater	Pollutants toxic to freshwater ecosystems
Nuclear waste	Radioactive waste
Ecotoxicity to land	Pollutants toxic to terrestrial ecosystems
Waste disposal	Material sent to landfill of incineration
Fossil fuel depletion	Depletion of coal, oil or gas reserves
Eutrophication	Water pollutants promoting algal blooms
Photochemical ozone creation	Air pollutants producing low-level zone
Acidification	Emissions causing acid rain

Table 1: Environmental impact categories used by BRE in Green Guide to Specification

The production of the LCAs of the products listed in the *Green Guide to Specification* (BRE 2006) hence started with an LCI aimed at assessing the quantities of each of the 13 aspects presented above, produced or consumed during each phase of the product life-cycle.

1.3.3 Defining environmental impact

From these environmental impacts, a single scoring can be extracted which represents the environmental impact of the product. This comprises two phases (Anderson 2009):

- Characterisation: Once the LCI is completed, a '*characterisation*' phase follows, during which each resource/emission is assigned to one or more of the environmental impacts considered,
- Normalisation: Finally the impacts can be '*normalised*', using standard values and converted into a single rating summarising the environmental impact of the building / product through a series of weighting factors representative of the relative importance given to each environmental impact.

This methodology provides one possible definition of the embodied environmental impact of a product or building.

This section reviewed one possible definition of the embodied environmental impact of a product or building. It highlights the challenges and uncertainties associated with the exercise. The example of the ICE database (Hammond 2011) highlights the need to use the data from general reviews of the environmental impacts associated with a product or a material with care when making design decisions based on them.

The review of a building's life-cycle also highlighted possible routes to reduce the environmental impact of building structures. These are reviewed in more detail in the following section.

1.4 Options for reducing the environmental impact of building structures

Previous sections have focused on buildings as single entities. From this section onwards, the focus is narrowed down to one of the key components of buildings, and the main topic of this thesis: their structure.

Data on the significance of the environmental impact of structures as a proportion of that of whole buildings is relatively scarce. A recent study of a building located at One Kingdom Street, Paddington Central in West London, concluded that 57% of all the initial embodied carbon in the building is in items specified by the structural engineer (dCarbon8 2007). Despite its focus on carbon only this study indicates that structural engineers have a potentially important role to play in reducing the environmental impact of buildings.

Looking at the diagram of a building's life-cycle presented in Figure 6, several options to reduce the embodied environmental impact of buildings can be identified. In a previous study (Thirion 2010), the author identified six main possible strategies organised in two main groups as summarised in Figure 7: *Life-time structural engineering* and *Eco-efficiency of structures*.

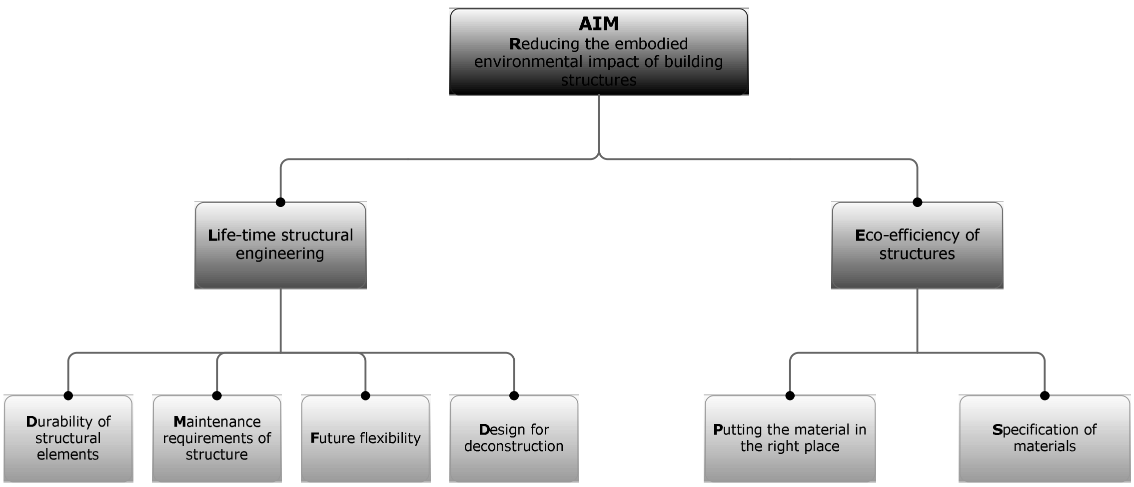

Figure 7: Diagrammatic representation of possible areas of actions to reduce the embodied environmental impact of structures

1.4.1 Life-time structural engineering

Measures falling under the *life-time structural engineering* heading focus on what happens to building structures after completion. The focus of such measures is beyond initial construction and considers instead the whole life requirements of structures.

Measures in this group include:

- *Durability of structures and maintenance requirements*: Finding an adequate balance between the need to maintain and replace structural elements over the life-span of a building, and the implications on the environmental impact of the structure of increasing their durability (Landolfo 2010).
- *Future flexibility*: Providing the structure of a building with characteristics such that it can fulfil its purpose despite the changing requirements from its occupants (Kincaid 2002, Brant 1995).
- *Design for deconstruction*: Designing the structure of a building to maximise the potential for components to be reused and materials to be recycled at the end of its life (Crowther 1999, Durmisevic 2006).

1.4.2 Eco-efficiency of structures

The *eco-efficiency* heading covers all measures which result in a reduction in the initial embodied environmental impact of structures.

This includes the specification of materials of reduced embodied environmental impact, which is the topic of most of the work done on the sustainability of structures, both in structural and architectural practices, and in research.

Specification of materials comprises two main threads:

- Reducing the environmental impact of currently mainstream construction materials: by using less environmentally damaging replacement products, like cement replacement products in concrete, or by increasing the recycled content of the materials, by using recycled aggregates in concrete for example.
- Building with currently alternative materials: buildings using currently alternative materials such as rammed earth, hempcrete or straw bale are often showcased as the answer to the development of sustainable building structures. They often face issues beyond their technical viability and more related to their acceptance by the wider public. These issues need to be addressed before these materials can become a serious option to compete with conventional materials.

The second set of measures related to eco-efficiency of structures is what is called here '*Putting the material in the right place*' or in other words, *how given structural materials can be used in order to reduce the initial embodied environmental impact of building structures*.

Here it is worth commenting on the reason why this range of measures is not simply referred to as '*Reducing the quantities of materials used in structures'*. Structures using less material are obviously desirable in the context of reducing their embodied environmental impact, as a reduction in material usage results in a direct reduction in embodied environmental impact. But for structures using composite materials, other options exist beyond the mere absolute reduction of material quantities. Indeed, for structural elements using such materials, the relative quantities of the constitutive materials may be varied while leaving the capacity and stiffness of the element unchanged, but with possibly an impact on their embodied environmental impact.

It is to account for these measures that the term '*Putting the material in the right place*' is used here. The range of measures covered under this term is broad, and their review is the topic of the next section.

1.5 OPTIONS FOR 'PUTTING THE MATERIAL IN THE RIGHT PLACE'

In a previous study of the structural engineering design process (Thirion 2010), the author has reviewed the possible options for reducing the initial embodied environmental impact of building structures through the sustainable use of structural materials. From this review, three main areas of investigation are proposed here:

- Environmentally-conscious design practice,
- Refined design criteria and design standards,
- Technological change.

These three aspects are reviewed in turn in the following paragraphs.

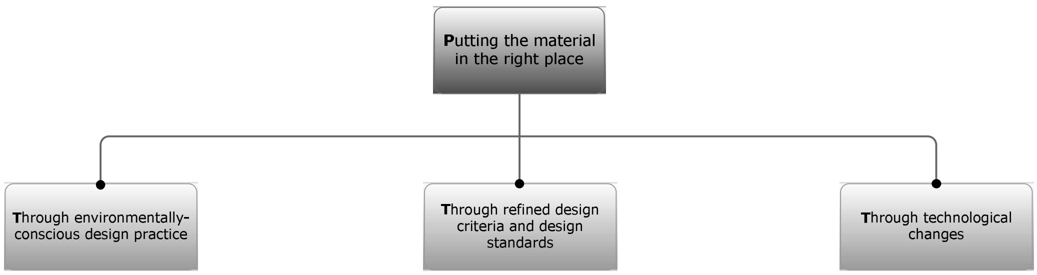

Figure 8: Proposed areas of investigation to reduce the initial embodied environmental impact of building structures through the sustainable use of structural materials

1.5.1 Environmentally-conscious standard practice

As explained in Section 1.2.2, UK building regulations currently focus only on operational aspects. Aspects relating to the embodied environmental impact of buildings are left unconsidered. There is hence no legislated incentive for designers to reduce the impact of the structures they design. It is hence reasonable to assume that building structures are designed and built with a greater embodied environmental impact than could theoretically be achieved by making the most of the design, fabrication and construction methods and technology commonly available today.

The forensic review by the author (Thirion 2010) of an existing building has highlighted a number of aspects which confirmed this assumption. Regarding the design loading considered for example, it was found that the reduction in the imposed loads seen by the supporting structure – columns and foundations – of a multi-storey building like that reviewed, which is permitted by the code of practice used in the design of the building, was found not to have been considered. For the five-storey building under consideration, this reduction is, according to BS6399 (1996, clause 6.2), of 40% for the foundations and basement columns.

Another aspect highlighted concerned the design of the columns, which was found to be based on a one-size fits all scenario: The original design only used a limited number of section types throughout the building, thus resulting in a significantly low rate of work for most of the columns in the structure.

These two examples relate to the calculation and detailing of the structure. At an earlier stage of the design process, the choice of a particular structural system can have a significant impact on the initial embodied environmental impact of a structure. Griffin (2010) has carried out a comparison of the quantities of embodied carbon in six commonly used structural flooring systems – flat slabs, flat slabs with drops, one-way spanning slabs on beams, waffle slabs and composite slabs and steel beams using a steel decking. The study demonstrated that savings of between 17% and 55% could be obtained in the embodied energy of floors by appropriately selecting the type of structural system used.

At a level higher again, the global shape and topology of a structure can have a significant impact on the quantities of materials required as demonstrated by the example of the Olympic Velodrome as summarised by Allwood (2012).

These examples illustrate the type of measures considered in this category. These measures do not aim at challenging current knowledge and abilities. Instead, they focus on the theoretical impact that could be obtained from a more environmentally-conscious approach to structural design, where all opportunities given by existing codes of practice, fabrication and construction methods are implemented to reduce the initial embodied environmental impact of structures.

1.5.2 Refined design criteria and design standards

The second category covers the choice of design criteria and the refinement of structural design standards.

Structural design relies on the use of a number of design criteria, which are effectively specifications of the required performance of structures. They consequently have a direct impact on the required strength and stiffness of structures and on the quantities of materials to be provided.

A key design criterion is design loading. Designing a structure for the level of loading adequate for its purpose is an important aspect in the design of material-efficient structures, as explained in the IStructE guide on sustainable construction in the chapter regarding specific tasks for the engineer (IStructE 1999, p.33). The author (Thirion 2010, pp.55-56) previously reviewed two studies which are critical of the loading used in the design of buildings.

The '*Stanhope study*' (Fitzpatrick 1992) was carried out in the early 1990s and criticised the then common tendency to design for higher imposed loads than required by codes of practice. According to the authors, this was due to a perception in the market place that the resulting extra capacity would make buildings designed to such loads attractive to a wider group of potential investors. The lasting occurrence of this practice is confirmed by a more recent article by Davis Langdon (2004) which states that "*some developers specify higher loads throughout [a building] to increase the flexibility of the floor plates*".

However, the Stanhope study argues, following the review of a particular building, that "*it must be realised […] that only 1% approximately of the floor area [of the building] has needed a loading capacity in excess of the code threshold*".
This quote points out the interaction between flexibility of use and quantities of material required, and highlights the necessity to develop a performance-based approach to flexibility of use to ensure that the extra capacity provided for the purpose of increasing future flexibility is actually used during the lifespan of the building.

Another interesting study regarding design loading is that carried out by Alexander (2002). The study, based on a literature review, questions the approach taken by building codes regarding imposed loading. Alexander argues that the imposed loading value used should vary with the size of the bays. He also describes designing continuous members for pattern loading as highly over-conservative as the probability of occurrence of such a configuration is extremely low.

These two studies are critical of the loading used in the design of buildings: one of design loading used in practice and the other of that proposed by codes of practice. Both consider it over-conservative, and consequently resulting in structural members with unnecessary over-capacity.

Serviceability (SLS) criteria are another important design criterion. SLS criteria are limits placed on the deflections of structural elements and the deflections seen by building structures, in order to ensure that structures are useable and provide the appropriate level of comfort to their users. Indicative limiting values are given by codes of practice, which require that the values used be discussed and agreed with the client and the rest of the design team on a case-by-case basis. However, in most cases, default values given by the codes are taken to be complied with, without considering which maximum values could actually be acceptable given the situation under consideration.

The issue of SLS criteria has multiple facets among which the users' comfort and the potential damage to the finishes supported by the structure. It is also indirectly linked to dynamics, as the standard values proposed by the codes are such that they typically ensure that vibration aspects are not an issue.

These SLS criteria could be reassessed to lead to lighter structures without affecting their fitness for purpose. To focus on the avoidance of damage to finishes for example, the limiting values specified by BS 8110 regarding the deflections in a flexural element after the construction of fragile or non-fragile finishes is of 20mm whatever the span of the member (BSI 1985, p.13). This requirement is to do with ensuring that finishes do not get damaged. Effectively building structures are designed not to move when an alternative would be to allow them to move more, and to detail the finishes to cope with these movements. This alternative approach may result in significant material savings.
The same is true of the comfort of users, as the limits set by the codes seem to be more historical than experimental and could hence probably be reduced.

The two above examples show that there may be some potential to reassess the design criteria used in the design of building structures. Another aspect which may be considered is the level of sophistication of codes of practice. Gardner (2007) reports for example that the level of refinement of the design standards related to the design of carbon steel structures is greater than that of both aluminium and stainless steel structures, due to the greater amount of data available and more research having been carried out. Refining the design codes for these two materials may result in a reduction in the quantities of materials required in structures using them.

These examples illustrate the range of aspects considered in the second category proposed in Section 1.5, and which could result in a reduction in the material quantities required in building structures.

1.5.3 Technology

The last category proposed to reduce the initial embodied environmental impact of building structures through the sustainable use of structural materials is termed '*technology*'. It is arguably the broadest of all three, and covers the development of innovative structural systems as well as new design, fabrication and construction techniques.

The recent work done at Bath University on fabric-formed structures is used as an example of the measures in this category. Starting from the realisation that most of the material in structural elements of constant sections works below its capacity, the team investigated the potential to form reinforced concrete beams of varying sections by using a formwork made of fabric. For a particular beam, Ibell (2007) demonstrated that savings of at least 35% could be obtained in the amount of concrete required by using such a technique.

This review of the possible areas of investigation to reduce the initial embodied environmental impact of building structures through the sustainable use of structural materials concludes the first chapter of this thesis. This chapter was the opportunity to discuss the contribution of the built environment to current environmental issues and the growing significance of embodied aspects, to review how the environmental impact of buildings or products can be defined and assessed, and the challenges associated with such an endeavour. Opportunities to reduce the embodied environmental impact of building structures were highlighted and a classification proposed. A framework was subsequently proposed for the investigation of the main topic of this thesis.

The following four chapters present two investigations carried out on the topic of reducing the initial embodied environmental impact of building structures through the sustainable use of structural materials. They respectively relate to the third and first areas of investigation described above.

- The first study investigates the potential for greater material-efficiency in the most commonly used structural element type in steel structures: steel I-beams.
- The second study presents an investigation into the low embodied carbon design of concrete flat slabs.

Chapter 2
Shape optimisation of steel I-beams: Development of theory

Figure 9: Nomenclature of variables used in Chapters 2 and 3

$W, weight\ of\ beam.$

$l, span\ of\ beam.$

$N, number\ of\ segments\ used\ to\ discretise\ the\ beam.$

$t_i, T_i, D_i, B_i, section\ dimensions\ in\ segment\ i, i \in [\![1; N]\!].$

$I_i, second\ moment\ of\ area\ of\ section\ in\ segment\ i, i \in [\![1; N]\!].$

$f_y, steel\ yield\ strength.$

$E, steel\ Young's\ modulus. Taken\ as\ 210GPa\ (Clause\ 3.2.6(1) of\ EN\ 1993-1-1 (BSI\ 2005).$

$\delta, beam\ deflections\ at\ midspan.$

$\delta_{QP}, \delta_{IL}, beam\ deflections\ at\ midspan\ under\ quasi - permanent\ and\ imposed\ loads\ respectively.$

$K_i, section\ curvature\ at\ middle\ of\ segment\ i, i \in [\![1; N]\!].$

$\theta_i, angle\ of\ rotation\ of\ section\ at\ middle\ of\ segment\ i, i \in [\![1; N]\!].$

$u_i, vertical\ displacement\ at\ middle\ of\ segment\ i, i \in [\![1; N]\!].$

$M_{Ed,i}, V_{Ed,i}, design\ bending\ moment\ and\ shear\ force\ at\ middle\ of\ segment\ i, i \in [\![1; N]\!].$

$M_{Rd,i}, V_{Rd,i}, design\ resistance\ to\ bending\ moment\ and\ shear\ force\ of\ section\ in\ segment\ i, i \in [\![1; N]\!].$

$M_{el,Rd,i}, design\ elastic\ resistance\ to\ bending\ moment\ of\ section\ in\ segment\ i, i \in [\![1; N]\!].$

$M_{pl,Rd,i}, design\ plastic\ resistance\ to\ bending\ moment\ of\ section\ in\ segment\ i, i \in [\![1; N]\!].$

$M_{f,Rd,i}, design\ plastic\ resistance\ to\ bending\ moment\ of\ the\ section\ consisting\ of\ the\ flanges\ only\ in\ segment\ i, i \in [\![1; N]\!].$

$V_{bw,Rd,i}, contribution\ of\ the\ web\ to\ the\ design\ resistance\ for\ shear\ of\ section\ in\ segment\ i, i \in [\![1; N]\!].$

$V_{bf,Rd,i}, contribution\ of\ the\ flanges\ to\ the\ design\ resistance\ for\ shear\ of\ section\ in\ segment\ i, i \in [\![1; N]\!].$

This is the first of two chapters dedicated to the study of the shape optimisation of steel I-beams. The study relates to the third area of investigation highlighted in the last section of Chapter 1, as it investigates the potential for reducing the initial embodied environmental impact of building structures through the introduction of a new technology: rolled steel I-beams with a section which varies along their length.

This chapter introduces the research question, as well as the methodology and theory used to investigate it.

2.1 INTRODUCTION

In this section, a review of the constraints and drivers affecting material usage in structures and structural elements is presented, which serves as an introduction to the research question which follows.

2.1.1 Constraints and drivers of material-efficiency

Today, most structural elements used in typical buildings have the potential to be made more material-efficient. In other words, they could be shaped to use less material while fulfilling the same function: carrying the same loads, offering identical levels of comfort to occupants, and guaranteeing that deformations are kept below acceptable limits for the finishes that they support.

Indeed, the choice of their shape is often driven by a desire to simplify the fabrication and construction processes involved in their making rather than to minimise the quantities of material they use. Beams provide a meaningful example of this situation: they are commonly given a constant section along their length, while the resistance and stiffness required vary along the length. Their section could hence be made to vary to exactly match the resistance and stiffness required.

Considering the simplest example of the ULS design of a simply-supported beam under a uniformly distributed load, the bending moment in the element follows a parabola and is maximal at midspan, while the shear force varies linearly and is maximal at the supports. The beam may hence be shaped to provide the bending and shear resistances strictly required at each point along the beam. An indicative profile of what such a beam may look like is shown in Figure 10.

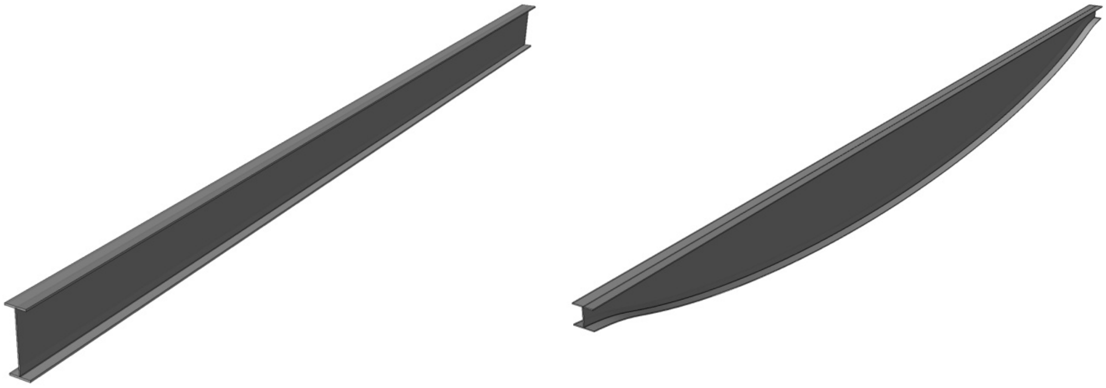

Figure 10: Constant and variable section beams

Attempting to minimise the quantities of material used in a structural element or a structure by shaping it has guided the work of engineers throughout history. A structure as old as the Pantheon, in Rome, displays a shell varying both in thickness and density along its span. As interestingly, the first successful attempt to predict theoretically the minimum amount of material required in a structural beam of varying section immediately followed the first step from a structural design using geometry-based rules towards the present theory of statics. Galileo is responsible for both advances as described in his *Dialogues concerning two new sciences* (Galileo 1638).

The structural problem at the heart of the dialogues is that of a timber pole with one end fitted into a masonry wall and loaded at its free end, in other words a cantilever beam, as represented on Figure 11a. Galileo set himself the task of calculating the collapse load of such a cantilever of given cross-sectional dimension. As soon as he obtained his first results on a beam of constant section – out by a factor of 3, which was the source of much work among seventeenth and eighteenth century scientists (Heyman 1998) – Galileo started tackling the problem of determining the shape of a cantilever beam of equal resistance but reduced cross-section along its length, as illustrated on the diagrams in Figure 11b and Figure 11c.

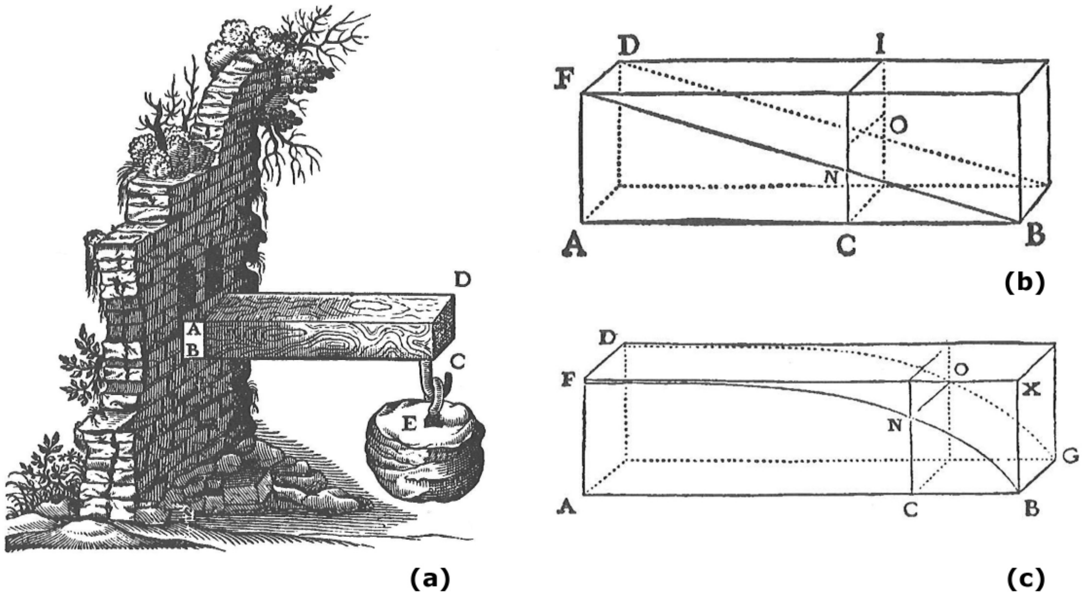

Figure 11a: Diagram of the cantilever beam problem considered in Galileo's dialogues (1638)

Figure 11b: First attempt at drawing a beam of constant width displaying an identical resistance to a vertical load applied at its tip at all cross-sections

Figure 11c: Second and successful attempt at drawing a beam of constant width displaying an identical resistance to a vertical load applied at its tip at all cross-sections: the beam depth follows a parabola

Today, almost four centuries later, our understanding of statics has evolved significantly, and the progresses made in the field of structural optimisation, coupled with the increase in computing power over the past 50 years, enable the assessment of the quantities of material required at each point in a structure very efficiently. But it is striking to notice that structures do not consistently make optimal use of the materials they are made of. This finds its explanation in the fact that the way in which materials are used in a structure is not only linked to available analysis capabilities, but depends on a much broader range of factors, some technical and some socio-economic.

Typically, the ratio between the cost of materials and the cost of labour is known to influence the structures produced: with cost of labour exceeding that of materials, structures tend to favour ease of fabrication and construction over attempting to minimise material quantities. On the contrary, when material cost outweighs labour cost, the structures developed aim to minimise material quantities, which often results in an increase in construction complexity and in the workforce required. The structures shown in Figure 12 and Figure 13 below, both by Uruguayan engineer Eladio Dieste, are examples of structures built in a context where material cost outweighs labour cost. Indeed, despite the significant construction complexity involved, these brick Gaussian vaults were economically viable for warehouses in 1960s Uruguay. These structures achieve extreme span-to-depth ratios and use an exceptionally limited amount of material, while the workforce required is significant.

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Figure 12: Construction of a Gaussian vault by E. Dieste (from Pedreschi 2000)

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Figure 13: Gaussian vaults under construction by E. Dieste (from Pedreschi 2000)

Today, the growing significance of the embodied environmental impact of buildings in their whole life-cycle impact, and the increasing importance given to environmental issues in general, constitute a new driver for attempting to reduce the quantities of materials used in structures. In fact, in the case of steel, reductions in material demand are indispensable for the industry to achieve the 2050 targets in reduction in carbon emissions recommended by the IPCC. As Alwood et al. (2012) point out, implementing the most advanced processes on all steel manufacturing plants worldwide would not be sufficient to counterbalance the anticipated rise in demand in steel products over the coming decades. Moreover, following decades of sustained research and development, these processes are already extremely energy-efficient and close to their theoretical thermodynamic limits. Tackling the demand for steel hence needs to be considered, and measures resulting in a reduction in the quantities of steel required for a specific task are an obvious option.

If commercial interests have driven improvements in the energy-efficiency of manufacturing processes, there have not been such incentives to encourage reductions in the quantities of material used in building structures. For steel structures there exist two main reasons why more material is used than is strictly required beyond the reasons linked to current practice, current codes of practice and commonly used design criteria. As previously highlighted with the example of beams, the shape of common structural steel elements is not optimal. Additionally, further rationalisation occurs as sections of structural members are commonly chosen from a finite set of standard sizes produced by manufacturers and listed in catalogues.

Addressing these issues is feasible but strongly limited by the availability of appropriate fabrication and construction methods which would make such products commercially viable.

2.1.2 Research question

The study presented in this chapter has been triggered by the work done by a leading rolling mill manufacturer who is developing a method to roll steel I-beams to order and with a cross section that may vary along the length of the element. This technology would hence address the two reasons discussed above for which steel structures use more material than strictly required: standardisation and shape. The present study is intended to provide an understanding of the geometry of the beams which such a rolling mill should target.

Previous studies have investigated the question of the beam profiles resulting in minimum weight elements. Pedersen and Pedersen (2009) proposed an analytical solution to this question for a range of cross-sectional shapes and design situations. These are however pure theoretical results and would not comply with current codes of practice as several practical issues such as local buckling of the elements of the section are deliberately left unconsidered. Carruth et al. (2011) have investigated the potential weight savings to be obtained by using a beam the depth of which varies along its length instead of a standard universal beam. The study, carried out to BS5950, reported savings in steel weight of as much as 30%. This study demonstrates the potential value of a mill able to roll varying section beams. However, it does not provide an understanding of the relative value of other possible geometries, which may indeed be easier to achieve, or may yield greater material savings.

In the development of such a technology, a balance needs to be found between the benefits to be obtained, here a reduction in the weight of steel used, and the complexity of the required rolling mill. This complexity is influenced by the number and type of dimensions required to vary in the beam, and in turn has a direct impact on the capital cost of the mill. The present study hence attempts to compare the relative merits of different beam geometries. The question asked is in fact threefold: if a bespoke section could be used instead of a catalogue section, what weight savings would be obtained? Also, if a single dimension was to be chosen to vary along the length of the beam, which one would produce the largest savings? And finally, are there any benefits from adding further complexity and allowing more than one dimension to vary at a time? The study is carried out to Eurocode 3 (BSI 2005, BSI 2005b) which is now the prevailing structural steel design code in Europe.

In the following section, the methodology proposed to investigate these questions is presented.

2.2 PROBLEM STATEMENT

This section characterizes the problem studied. The scope of the study is first described, followed by a presentation of the particular design configuration used for the investigation. A presentation of the structural design criteria considered concludes the section.

2.2.1 Description of study

The minimisation of the quantities of material used in a structure or in a structural element is a broad topic. Different levels of investigations exist depending on the level of constraint imposed on the range of possible solutions considered. Bendsøe (2003) identifies three main levels of investigation depending on the characteristics of the structure or the structural element which are allowed to vary. Figure 14 summarises these three levels for trusses and beams.

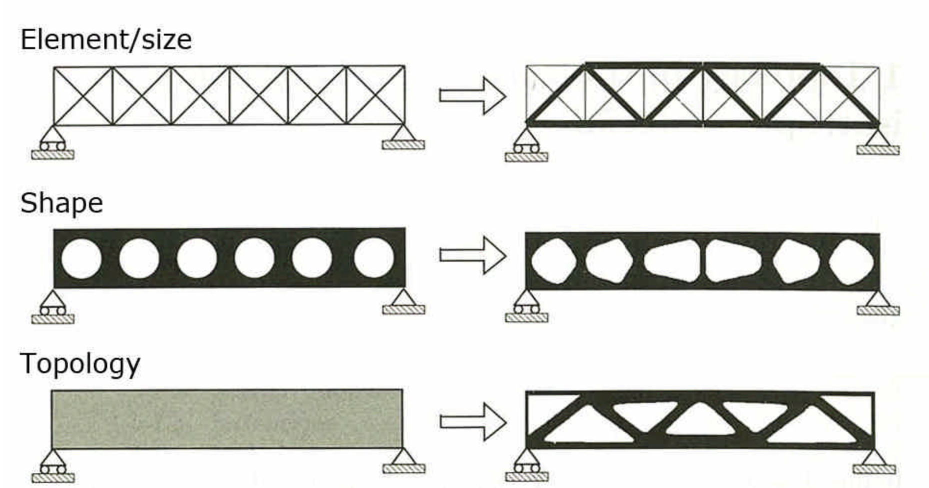


Figure 14: Three main levels of investigation in the study of the optimal shape of structures (after Bendsøe 2003)

The present study is intended to inform the development of a particular fabrication method. The range of solutions investigated are hence chosen in accordance with the possibilities presented by this method. As a consequence, the beams considered in this investigation use an I-section, and the thickness of the flanges and web, although they may vary along the length of the beam, are constant in each section. Additionally, the presence of holes in the web or the flanges is not considered. Opening the range of possible shapes would surely result in even lighter elements, but would not comply with the fabrication constraints considered here.

The section of an I-beam can be fully described by four dimensions: *t*, web thickness, *T*, flange thickness, *D*, section depth, and *B*, flange width, as shown in Figure 15. The study presented here looks into the relative weight savings obtainable from varying each of these four dimensions separately, as well as the further potential benefits of varying several dimensions at a time. The baseline against which these results are compared is the weight of a constant section beam using the lightest viable section available from a steel section supplier's catalogue (Tata Steel 2011). The results are also compared against the weight of a minimum weight constant section beam, the dimensions of which do not come from a supplier's catalogue, but are optimally chosen to produce the lightest beam possible for the design configuration considered. In the rest of this study, such a beam is referred to as a *constant customised section beam*. In total, 13 cases are investigated as summarised in Table 2.

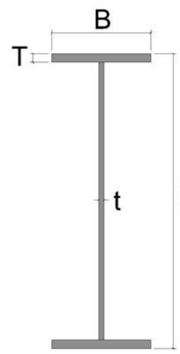


Figure 15: Dimensions defining an I-section

Analysis case number	Dimension(s) varying along beam length	Dimension(s) constant along beam length
0	none	t, T, D, B
1	T	t, D, B
2	D	t, T, B
3	B	t, T, D
4	t, D	T, B
5	T, D	t, B
6	T, B	t, D
7	D, B	t, T
8	t, T, D	B
9	t, T, B	D
10	t, D, B	T
11	T, D, B	t
12	t, T, D, B	none

Table 2: Analysis cases considered

2.2.2 Design configuration used for investigation

A particular design configuration needs to be selected to carry out the investigation.

2.2.2.1 Choice of building typology

The British Constructional Steelwork Association (BCSA) produces a yearly review of the UK consumption of constructional steelwork per sector of activity. Table 3 summarises their results for 2009 (BCSA 2009). The results highlight that infrastructure (power, bridges etc.) only represent a small proportion of the national consumption of steel. Based on their respective activities, the remaining sectors can be assumed to use two main building typologies principally: single storey portal frames principally for the industrial, agriculture and retail sectors, and multi-storey posts and beams construction for the office, leisure, health, education and domestic sectors. These two typologies consume the largest proportion of constructional steel in the UK.

The latter building typology is considered in this study. More particularly, the layout, dimensions and design loading of the design configuration used for investigation are chosen to be representative of office building design as detailed below.

Sector	UK steel consumption (thousand tonnes)
Industrial	280
Agriculture	45
Retail	80
Offices	105
Leisure	85
Health	29
Education	70
Domestic	19
Power	32
Bridges	28
Other infrastructure	23
Other	29
Fabricated imports	17
Total UK consumption	**842**

Table 3: Consumption of constructional steelwork per sector of activity

2.2.2.2 Structure considered

The structure considered for investigation is shown in Figure 16. It is a 9m by 9m bay with two primary beams, and secondary beams running orthogonally and spaced at 3m.

The study focuses on one of the secondary beams which are taken as simply-supported. Lateral-torsional buckling is ignored as the floor slab which the beams support is considered to provide a lateral restraint to the compression flange of the beams. Steel grade S355 is used.

One aspect makes the chosen design configuration not fully representative of office building design: in this study no composite action is considered between the beam and the slab. Composite floor systems are increasingly used. Carruth et al. (2011) report that up to 50% of all new building projects may use them. This could be investigated in another study.

2.2.2.3 Design loading

The loading intensities considered are consistent with the office building usage considered.

The total super-imposed dead load (SDL) corresponds to the weights of the elements listed in Table 4.

An imposed load (IL) of 2.5kN/m^2 is considered.

The self-weight of the beam itself is ignored on the basis that it would have a negligible impact on the result being given that it is an order of magnitude smaller than the other loads.

Two combinations are considered in the design of the slabs: they are described in Table 5 (BSI 2004, BSI 2006).

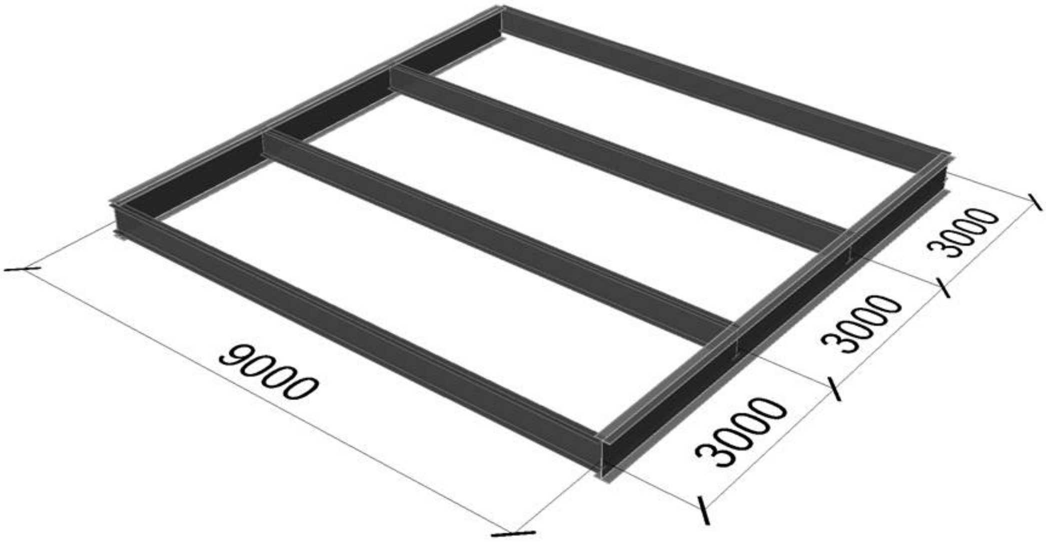


Figure 16 Design configuration used for investigation (dimensions in mm)

Element	Surface load (kN/m^2)
150mm RC slab	3.75
Screed (55mm thick)	1.30
Ceiling (finishes and services)	0.50
Partitions	1.00
Total	**6.55**

Table 4: Breakdown of super-imposed dead loads considered

ULS combination	1.35 SDL + 1.50 IL
SLS combination	1.00 SDL + 0.30 IL

Table 5: Load combinations considered

2.2.3 Structural design checks

As explained previously, this study is carried out to Eurocode 3 and the beams investigated must comply with the requirements from EN1993-1-1 and EN1993-1-5 (BSI 2005, BSI 2005b). This section describes the structural checks with which the beams must comply.

2.2.3.1 Deflections

To ensure comfort to occupants and prevent damage to the finishes supported by the floor, assumed here to be brittle, midspan deflections under quasi-permanent loads (SLS combination) and imposed loads are respectively limited to span/250 and span/360 (BSI 2008, clause NA 2.23).

2.2.3.2 Bending resistance

At any point along the length of the beam, the moment resistance of the section must be greater than the applied bending moment.

The bending resistance of a section depends on the slenderness of its flanges and web as this directly influences their local buckling resistances. EN1993-1-1 differentiates between four classes of sections which each can achieve a different bending resistance (BSI 2005, clauses 5.5 and 6.2.5). In the present study, only class 3 sections, which can develop their full elastic bending capacity, but cannot reach their plastic moment resistance, are considered. The fact that the increased moment resistance of classes 1 and 2 sections is not considered is further discussed in a later section in Chapter 3.

Allowing sections of the beam to become class 4 is likely result in greater savings than obtained when only considering class 3 section and possibly in a different repartition of the relative merits of allowing different subsets of the dimensions defining a section to vary. The relevant part of Eurocode 3 covering class 4 sections is EN1993-1-5 which is dedicated to the design of plated structural elements. The reason why class 4 sections were not accounted for here comes from the fact that EN1993-1-5 requires the use of Finite Element Modelling (FEM) to design class 4 section beams the section of which varies along the length (BSI 2005b, clause 2.5). The methodology adopted in this study does not include FEM. Another study would hence be required to carry out a similar study as presented here and including class 4 sections.

2.2.3.3 Shear resistance

Along the length of the beam, the shear resistance of each section must be greater than the applied shear force. In this study, the slenderness ratio of the web is allowed to go beyond the limit above which the shear buckling resistance of the web needs to be considered (BSI 2005, clause 6.2.6(6)). The relevant part of Eurocode 3 for this check is EN1993-1-5 (BSI 2005b). As explained above, this document covers the cases where the section of a beam is constant through formulae, but refers to the use of Finite Element Modelling (FEM) to investigate the cases where it varies along the length (BSI 2005b, clause 2.5). The methodology adopted in the present study does not include FEM which has the following consequences:

- For the cases where the web dimensions are constant – cases 0, 1, 3 and 6 in Table 1 – the web shear buckling resistance is accounted for as per EN1993-1-5 (BSI 2005b). Although strictly speaking the section varies as the flanges dimensions change along the length, the approach of the code remains valid as the maximum applied shear in the beam occurs at the ends of the beam where the flanges have the smallest cross-section.
- For the other cases, where the web dimensions vary, the results are given without web shear buckling resistance being accounted for. As a consequence, the weight savings given in this study for these cases are an upper-bound: the corresponding optimal beams are likely to weigh more than given here, as accounting for the shear buckling resistance of the web will impose an additional constraint.

For the cases where the depth of the beam varies, the change in angle in the tension flange of the beam along its length generates an additional compressive force in the web of the beam as explained by Owens (1989, p.18) and illustrated on Figure 17. The effect of this compression force is not taken into account in this study. Although this additional compression is expected to be small because the change in angle in the tension flange of the beams obtained is gentle, this is an additional reason why the weight savings given in this study for cases in which the depth of the beam varies are an upper-bound.

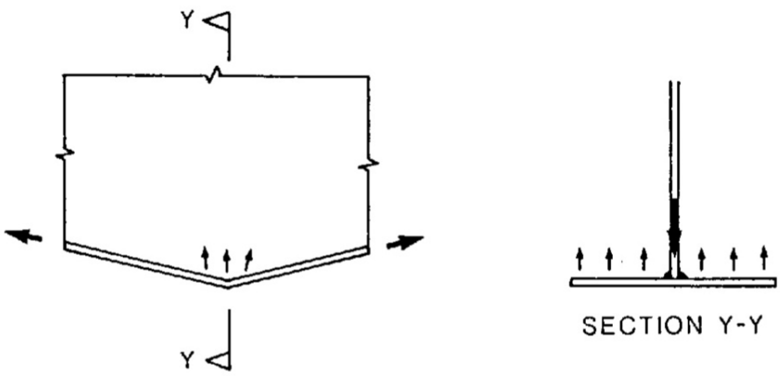


Figure 17: Additional compression force in web generated by change in angle in tension flange of I-beam

As will be seen in the presentation of the results, these limitations nevertheless enable reaching meaningful conclusions regarding which subsets of the dimensions defining the beam section yield the largest material savings when allowed to vary along the beam's length.

2.2.3.4 Flange induced buckling

Each section is checked to prevent the compression flange of the beam buckling in the plane of the web (BSI 2005b, clause 8.1).

2.2.3.5 Bearing area for slab

In order to ensure that sufficient bearing area is provided to the slab which sits on top of the beams, the Steel Construction Institute (SCI 2009, p.42) recommend that the top flange which supports the slab be at least 150mm wide. A lower limit of 150mm is hence placed on the width of the flanges of the beams.

This description of the structural design checks considered for the beams of this study concludes this section.

As explained previously, the aim of the study is to find the lightest beam which makes best use of the possible variations in a given subset of the four section dimensions along the length of the beam, and is viable for the design configuration considered. In effect, it is an optimisation problem in which the objective function is the weight of the beam which is sought to be minimised against a series of structural and geometric constraints. The implementation of this optimisation problem is the subject of the next section.

2.3 Optimisation

As explained by Papalambros and Wilde (2000), design optimisation involves the following aspects:

- *Parameterisation*, or the selection of a set of variables to describe the design,
- The *definition of an objective function*, expressed in terms of the design variables, which is sought to be minimised or maximised,
- The *definition of a set of constraints* which must be satisfied for a design to be acceptable. These are equality or inequality equations expressed in terms of the design variables,
- The *definition of allowed values for the design variables*.

In this section, each of these aspects is reviewed in turn. It is concluded by a discussion about the choice of algorithm used for this investigation.

2.3.1 Parameterisation

Since the system studied is symmetrical, only half of the length of the beam is modelled. To carry out the optimisation, the half-beam is discretised into N segments of equal length which are each assigned a constant section along their length, as shown in Figure 18. The four dimensions of each of these *N* I-sections are the *4xN* design variables of the optimisation problem. The section at segment number *i* is hence described by t_i, T_i, D_i and B_i.

Each design variable is allowed to vary between its limiting values independently from the values assigned to neighbouring sections. This way of parameterising the problem presents the benefit of avoiding over-constraining the range of possible solutions, compared, for example, with a parameterisation using a restricted family of shape functions to define the beam profile.

Indeed, a previous study by the author (Thirion 2010), which aimed to obtain a first assessment of the potential savings achievable by varying the section of beams along their length, a parameterisation using shape functions was used: both the flange width and the section depth were allowed to vary along the length in the form of parabolas. In the solution obtained, the depth of the beam at about a sixth of the length of the beam was found to be larger than strictly required. It was indeed constrained by the depth required to provide sufficient shear resistance at the support. The introduction of an additional point of inflection in the profile of the tension flange of the beam would have been required for the profile to be more optimal which was made impossible by the parameterisation chosen. The present study does not have this limitation.

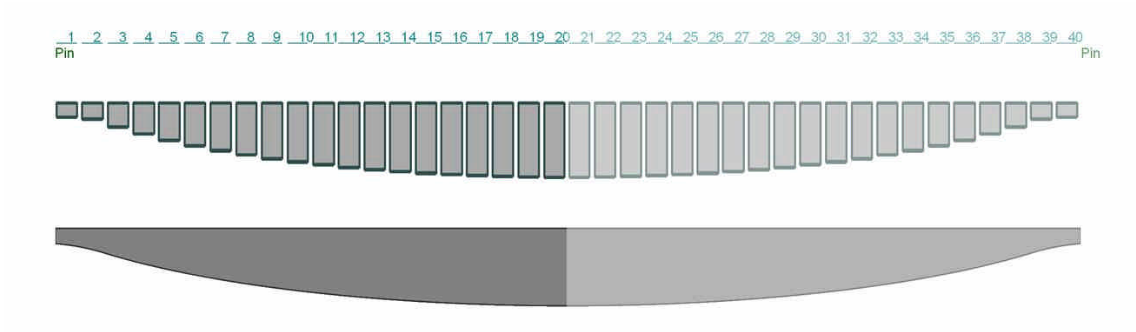


Figure 18: Discretisation of beam into constant section segments used for investigation

2.3.2 Objective function

The objective function of the optimisation problem is the weight of the beam which is sought to be minimised. It is calculated as follows:

$$W = 2 \times \frac{l}{2 \times N} \sum_{i=1}^{N} [t_i \times (D_i - 2 \times T_i) + 2 \times T_i \times B_i] \qquad \text{Eq. 2-1}$$

2.3.3 Structural constraints

The structural checks previously described in Section 2.2.3 form the structural constraints of the optimisation process, with which the beams must comply to be viable. This section summarises how they are derived mathematically.

2.3.3.1 Deflections

The beam deflections at midspan are calculated by performing a double integration of the curvatures using the trapezium rule. This involves three steps: the calculation of the curvature at the central point of each segment of the beam (Figure 18), of the angle at each point and finally of the vertical displacement at each point.

2.3.3.1.1 Curvatures

Curvatures are first calculated at each sampling point along the length of the beam as follows.

$$\forall\, i \in [\![1;N]\!],\, K_i = \frac{M_{Ed,i}}{EI_i} \qquad \text{Eq. 2-2}$$

$$K_{midspan} = \frac{M_{Ed,midspan}}{EI_N} \qquad \text{Eq. 2-3}$$

2.3.3.1.2 Angles

The angles that the tangent to the deformed neutral axis forms to the undeformed neutral axis at the central point of each segment are then calculated as follows.

$$\theta_N = \frac{K_N + K_{midspan}}{2} \times \frac{l}{4N} \qquad \text{Eq. 2-4}$$

$$\forall\, i \in [\![1;N\text{-}1]\!],\, \theta_i = \theta_{i+1} + \frac{K_i + K_{i+1}}{2} \times \frac{l}{2N} \qquad \text{Eq. 2-5}$$

$$\theta_0 = \theta_1 + \frac{K_1}{2} \times \frac{l}{4N} \qquad \text{Eq. 2-6}$$

2.3.3.1.3 Vertical displacements

From these, the vertical displacement at each sampling point can then be calculated as follows.

$$u_1 = -\frac{\theta_0 + \theta_1}{2} \times \frac{l}{4N} \qquad \text{Eq. 2-7}$$

$$\forall\, i \in [\![2;N]\!], u_i = u_{i-1} - \frac{\theta_{i-1} + \theta_i}{2} \times \frac{l}{2N} \qquad \text{Eq. 2-8}$$

$$\delta = u_N - \frac{\theta_N}{2} \times \frac{l}{4N} \qquad \text{Eq. 2-9}$$

Another option to calculate the vertical displacements at midspan would have been to apply the principle of virtual work.

2.3.3.2 Local buckling

For the class of sections considered, EN1993-1-1 (2005) deals with local buckling by imposing limits of the slenderness ratios of the web and the flanges. According to clauses 5.5 of EN1993-1-1 (2005), the two following relationships must be verified to avoid local buckling of the web (Eq. 2-10) and of the flanges (Eq. 2-11) in class 3 sections.

$$\frac{D_i - 2 \times T_i}{t_i} \leq 124 \times \sqrt{\frac{235}{f_y}}, \forall\, i \in [\![1;N]\!] \qquad \text{Eq. 2-10}$$

$$\frac{B_i - t_i}{2 \times T_i} \leq 14 \times \sqrt{\frac{235}{f_y}}, \forall\, i \in [\![1;N]\!] \qquad \text{Eq. 2-11}$$

2.3.3.3 Bending resistance

As explained previously, two cases are considered regarding the shear resistance of the beams: one where web shear buckling is taken into account and one where it is ignored. This has an implication for the moment resistance which is calculated differently depending on the case considered.

2.3.3.3.1 Cases where web shear buckling is not accounted for

In the case where web shear buckling is not accounted for, the moment resistance is calculated according to clauses 6.2.5(2) and 6.2.8(3) of EN1993-1-1 (BSI 2005).

$$if\ V_{Ed,i} \leq 0.5 V_{Rd,i},\ M_{Rd,i} = M_{el,Rd,i}, \forall\, i \in [\![1;N]\!] \qquad \text{Eq. 2-12}$$

$$else,\ M_{Rd,i} = M_{el,Rd,i} \left[1 - \left(\frac{2\, V_{Ed,i}}{V_{Rd,i}} - 1\right)^2\right], \forall\, i \in [\![1;N]\!] \qquad \text{Eq. 2-13}$$

2.3.3.3.2 Cases where web shear buckling is accounted for

In the case where web shear buckling is accounted for, the moment resistance is calculated according to clause 7.1 of EN1993-1-5 (BSI 2005b).

$$if\ V_{Ed,i} \leq 0.5 V_{Rd,i},\ M_{Rd,i} = M_{el,Rd,i}, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-14}$$

$$else,\ M_{Rd,i} = M_{pl,Rd,i} - \left(M_{pl,Rd,i} - M_{f,Rd,i}\right)\left(\frac{2\,V_{Ed,i}}{V_{Rd,i}} - 1\right)^2 \leq M_{el,Rd,i}, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-15}$$

2.3.3.4 Shear resistance

As explained previously, two cases are considered regarding shear resistance: one where web shear buckling is taken into account and one where it is ignored. The calculation of the shear resistance in each case is presented hereafter.

2.3.3.4.1 Cases where web shear buckling is not accounted for

In the case where web shear buckling is not accounted for, the shear resistance is calculated according to clauses 6.2.6(2) of EN1993-1-1 (BSI 2005).

$$V_{Rd,i} = \frac{D_i \times t_i \times f_y}{\sqrt{3}}, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-16}$$

2.3.3.4.2 Cases where web shear buckling is accounted for

In the case where web shear buckling is accounted for, the shear resistance is calculated according to clauses 5.2(1) of EN1993-1-5 (BSI 2005b).

$$V_{Rd,i} = V_{bw,i} + V_{bf,i} \leq \frac{(D_i - 2\,T_i) \times t_i \times f_y}{\sqrt{3}}, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-17}$$

2.3.3.4.2.1 Contribution from the web to shear resistance of section

$V_{bw,i}$ represents the contribution from the web to the shear resistance of the section. It depends on a slenderness parameter $\bar{\lambda}_w$ defined in clause 5.3(3) of EN1993-1-5 (BSI 2005b) and calculated as follows.

$$\bar{\lambda}_{w,i} = \frac{D_i - 2 \times T_i}{86.4 \times t_i \times \varepsilon}, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-18}$$

The contribution from the web to the shear buckling resistance depends on a factor $\chi_w$defined in clause 5.3(1) of EN1993-1-5 (BSI 2005b) and calculated as follows.

$$if\ \bar{\lambda}_{w,i} \leq 0.83,\ \chi_{w,i} = 1, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-19}$$

$$else,\ \chi_{w,i} = \frac{0.83}{\bar{\lambda}_{w,i}}, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-20}$$

From this, the contribution from the web to the shear resistance of the section can be calculated as per clause 5.2(1) of EN1993-1-5 (BSI 2005b).

$$V_{bw,i} = \frac{\chi_{w,i} \times (D_i - 2 \times T_i) \times t_i \times f_y}{\sqrt{3}}, \forall\ i \in [\![1;N]\!] \quad \text{Eq. 2-21}$$

2.3.3.4.2.2 Contribution from the flanges to shear resistance of section

The contribution from the flanges to the shear resistance of the section is calculated as per clause 5.4(1) of EN1993-1-5.

$$V_{bf,i}=\frac{B_i \times {T_i}^2 \times f_y}{c_i} \times \left[1-\left(\frac{M_{Ed,i}}{M_{f,Rd,i}}\right)^2\right], \forall\, i \in [\![1;N]\!] \qquad \text{Eq. 2-22}$$

$$where\ c_i=l \times \left(0.25+\frac{1.6 \times B_i \times {T_i}^2 \times f_y}{t_i \times (D_i - 2 \times T_i)^2 \times f_y}\right), \forall\, i \in [\![1;N]\!] \qquad \text{Eq. 2-23}$$

2.3.3.5 Flange induced buckling

To prevent the compression flange of the beam from buckling in the plane of the web, the following equation must be verified as per clause 8 of EN1993-1-5.

$$\frac{D_i - 2 \times T_i}{t_i} \le 0.55 \times \frac{E}{f_y} \times \sqrt{\frac{(D_i - 2 \times T_i) \times t_i}{B_i \times T_i}}, \forall\, i \in [\![1;N]\!] \qquad \text{Eq. 2-24}$$

This constraint is not included in the optimisation process. Instead, compliance is checked on the results obtained from the optimisation process.

2.3.4 Geometrical constraints

In addition to the structural constraints described above, a series of geometric constraints are placed on the section dimensions in order to ensure that rational shapes emerge from the optimisation process. These are as follows:

- Geometrical constraint 1: At each section, the flange width B_i should remain greater than the thickness of the web, t_i.
- Geometrical constraint 2: At each section, the flange thickness T_i should not exceed half of the section depth, D_i.

An additional constraint is placed on the aspect ratio of the flanges to ensure buildability:

- Geometrical constraint 3: At each section, the flange width B_i should not be less than $5 \times T_i$.

2.3.5 Allowed values for design variables

As explained previously, a lower limit of 150mm is placed on the flange width in order to ensure that sufficient bearing area is provided for the slab supported by the beams (SCI 2009). The only constraint imposed on the other design variables is that they obviously need to remain positive.

2.3.6 Optimisation algorithm

In this section, the choice of algorithm used for the investigation is discussed along with its implementation.

2.3.6.1 Choice of algorithm

There exists a broad range of optimisation techniques and corresponding algorithms. These can be of two main types: *deterministic* methods and *stochastic* methods (Baldock 2007). Deterministic methods, as their name indicates, will always converge to the same solution for a given starting point, under fixed parameters. These methods may hence fail to find the global optimum of a problem presenting several local optima. Stochastic methods include an element of probability in the search for the optimal solution, and may converge to different solutions for a given starting point and fixed parameters.

Considering the case where the shear buckling resistance is not considered first, all the functions of the problem under consideration are monotonic: a reduction in any of the design variables defining a particular beam, t_i, T_i, D_i and B_i, for i between 1 and N, will result in a reduction in the weight of the beam, an increase in the midspan deflections through a reduction in the second moment of area of one section of the beam, and a reduction in the shear and bending resistances. This guarantees the uniqueness of the optimum in this case. A deterministic method is hence appropriate.

Moreover, the objective function, as well as the constraints, is generally smooth. All the functions describing the problem are continuous (C^0) and all are differentiable (C^1) apart from one: the constraint relating to the bending resistance does not have a continuous first derivative due to the dependency on the rate of work of the section, as made obvious in Equations Eq. 2-14 and Eq. 2-15. Despite this, the problem is overall smooth. And for smooth problems of this sort, gradient-based deterministic methods are appropriate. Moreover, as noted by Eiben (2002), on this type of problems, stochastic methods are generally found to underperform compared to deterministic methods.

The objective function and most of the constraints are non-linear functions of the design variables. The *Sequential Quadratic Programming* (SQP) algorithm is a gradient-based method designed to work on problems where the objective and constraint functions are multi-variable non-linear functions which are continuous (C^0) and have continuous first derivatives(C^1) (Matlab 2012). It is a robust and proven algorithm and is widely used for this type of problems as exemplified by the work of Katsikadelis (2005) and Carruth et al. (2011). As previously explained, the functions of the problem considered do not fully comply with the requirement for functions to be class C^1 as one of the constraint function is not strictly C^1. Non-compliance with the rule is however very localized and was not thought to have the potential to compromise the application of the algorithm to converge. This was indeed confirmed through its application.

In the case where the shear buckling resistance of the web is accounted for, some of the constraints functions do not comply with the characteristics discussed above. In particular, a larger number of functions do not comply with the requirement for functions to be C^1 and the monotonicity of the functions is less obvious. Consequently, the obtained result may hence depend on the starting point used to run the algorithm. To ensure that the obtained solution is close to the global optimum, the results obtained from running the analysis without considering the shear buckling resistance of the web are used as starting points of the analyses where it is considered, as these are known to be reliable starting points. As will be seen, the results obtained when accounting for the shear buckling resistance of the web are all found to be very close to those obtained when it is not accounted for, meaning that, if the results obtained are not global optima, they are very close to being.

2.3.6.2 Implementing the algorithm

The algorithm is implemented in Matlab. The function *fmincon* which run an SQP algorithm, and is available from Matlab's optimisation toolbox, is used. This takes as an argument the objective function, each of the constraints, as well as any limitation imposed on the values of the design variables. The algorithm developed returns the dimensions of the section of each of the 20 segments into which the beam is discretised, the weight of the beam found to be optimal, and the constraints which are *active* for the solution obtained. Active constraints are constraints which the solution is just about to violate. In other words, these are constraints with which the obtained solution would not comply if any of the design variables was to be modified. As will be seen in a later section, these are useful in the interpretation of the results returned by the algorithm.

Figure 19 gives a summary of the algorithm developed.

Figure 19: Summary of algorithm

Minimise: $W = 2 \times \frac{l}{2\times N} \sum_{i=1}^{N} [t_i \times D_i + 2 \times T_i \times B_i]$

Subject to:

- *Deflection constraints:* $\delta_{QP} \leq l/250$, $\delta_{IL} \leq l/360$
- *Resistance constraints:* $M_{Ed,i} \leq M_{Rd,i}$, $V_{Ed,i} \leq V_{Rd,i}$, $\forall\ i \in [\![1;N]\!]$
- *Local buckling:* $(D_i\text{-}2\times T_i)/t_i \leq 124\times\sqrt{235/f_y}, \forall\ i \in [\![1;N]\!]$

  $(B_i\text{-}t_i)/2T_i \leq 14\times\sqrt{235/f_y}, \forall\ i \in [\![1;N]\!]$
- *Geometric constraints:* $t_i \leq B_i$, $T_i \leq D_i/2$, $\forall\ i \in [\![1;N]\!]$

  $x_{min} \leq x_i \leq x_{max}$, $\forall\ i \in [\![1;N]\!]$ *for* x=t, T, D *or* T

Where:

- *W, weight of beam,*
- *l, span of beam,*
- *N, number of points used to discretise the beam, taken as 20,*
- t_i, T_i, D_i *and* B_i *section dimensions at section i,* $i \in [\![1;N]\!]$,
- f_y, *steel yield strength,*
- E, *steel Young's modulus,*
- δ_{QP}, δ_{IL} , *beam deflections at midspan under respectively permanent and imposed loads,*
- $M_{Ed,i}$, $V_{Ed,i}$, *design bending moment and shear force at section i,* $i \in [\![1;N]\!]$,
- $M_{Rd,i}$, $V_{Rd,i}$, *design resistances to bending moment and shear force at section i,* $i \in [\![1;N]\!]$,
- x_{min}, x_{max}, *lower and upper bounds placed on section dimensions.*

2.4 TESTING THE ALGORITHM

To test the validity of the reasoning presented in Section 2.3.6.1 regarding the uniqueness of the solution, the algorithm was run from a number of different starting points and it was verified that the algorithm converged to the same solution each time.

Figure 20 and Figure 21 present the evolution of the steel weight as a function of the number of iterations performed by the algorithm, respectively for the case where the depth of the beam is allowed to vary, and that where the web and flange thicknesses and the flange width are allowed to vary, and for a number of different starting points. In both cases, the algorithm is found to converge to the same solution.

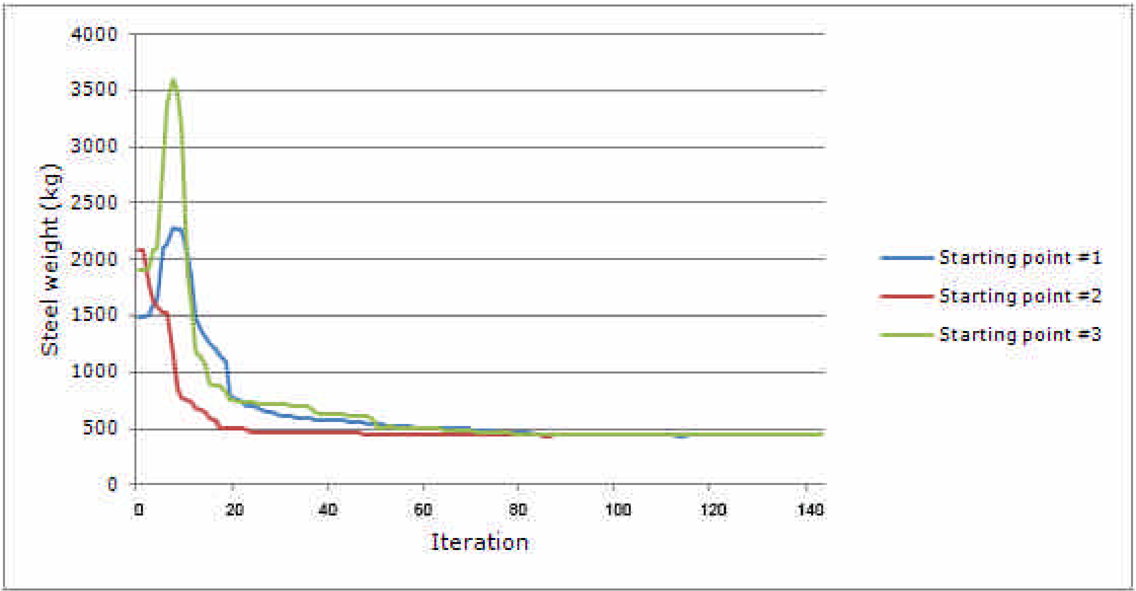


Figure 20: Evolution of steel weight as a function of numbers of iteration in optimisation process for three different starting points – Case where D varies

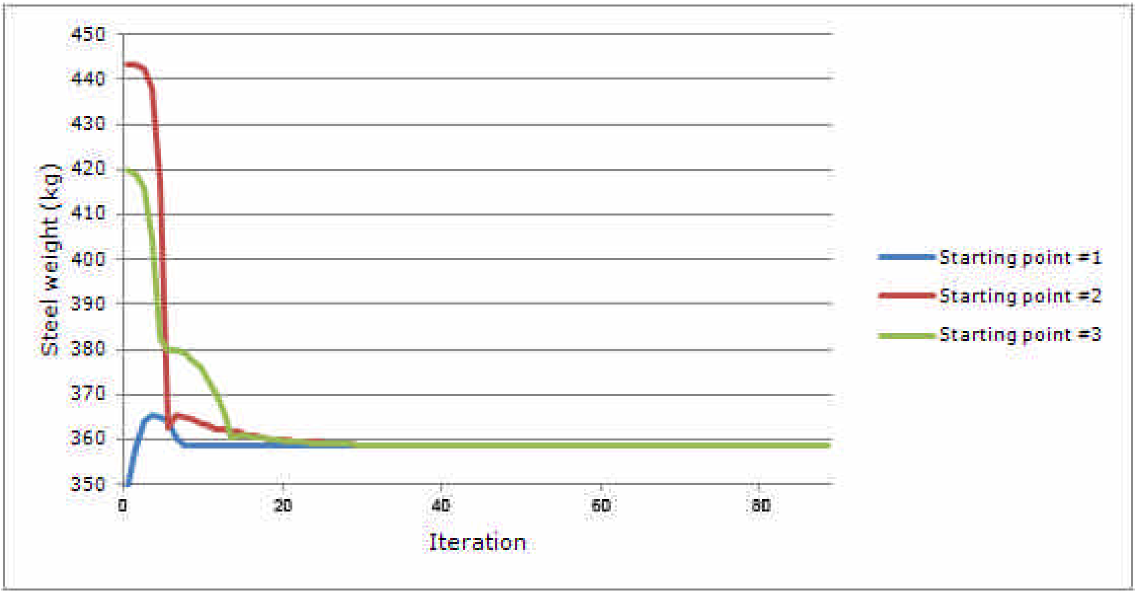


Figure 21: Evolution of steel weight as a function of number of iterations in optimisation process for three different starting point – Case where t, T and B vary

This concludes the first of the two chapters of this thesis dedicated to the study of the shape optimisation of steel I-beams. This chapter presented the research question considered as well as the methodology and theory used to investigate it. The next chapter is dedicated to the presentation, interpretation and discussion of the results obtained.

Chapter 3
Shape optimisation of steel I-beams: Results and sensitivity studies

3.1 INTRODUCTION

In this chapter, the methodology and theory presented in Chapter 2 are used to investigate the relationship between minimum weight optimised I-beams and complexity of fabrication. This involves allowing different subsets of the dimensions defining the section of an I-beam to vary along its length. Two sets of results are presented successively hereafter.

The first one relates to the case where the section depth *D* is unconstrained. In this case, *D* is allowed to become larger than the depth of the constant catalogue section beam used as a baseline for comparison.

The second set of results is motivated by the fact that, in the design of typical multi-storey office buildings, overall floor-to-ceiling height, and hence depth of the structural zone, is a key consideration, often sought to be minimised. As a consequence, in current practice, if a choice has to be made between a heavier shallower beam and a lighter but deeper one, the former is likely to be chosen in view of its other benefits on the design. The second set of results takes this into account through the introduction of an upper-bound on the allowable depth of the optimal beams.

3.2 RESULTS FOR UNCONSTRAINED SECTION DEPTH

In this section, the results obtained in the case where the section depth *D* is left unconstrained are presented.

3.2.1 Input parameters

The results presented in this section have been obtained by discretising the beam in 20 segments, i.e. $N=20$.

3.2.2 Profiles of beams

First, the profile obtained for the different analysis cases listed in Table 2 are presented and interpreted.

3.2.2.1 Results

The profiles of the dimensions varying along the length of the beam are presented in Figure 22a and Figure 22b. The exact dimensions are also given in a tabular form in Appendix B.

Figure 23 and Figure 26 respectively present a 3D view and an elevation of the beam obtained when the section depth is allowed to vary along the length.

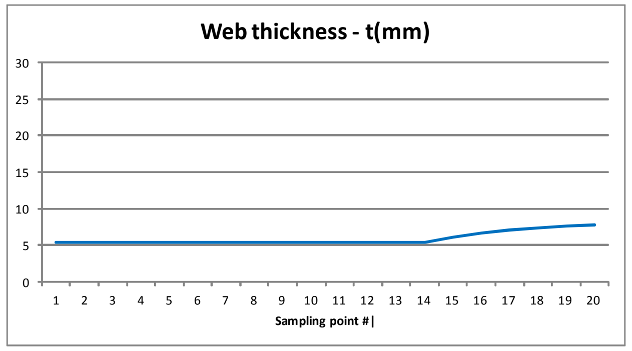
The lightest catalogue section found to be viable from a Tata Steel catalogue (Tata Steel 2011) is a UKB 457x152x60. The dimensions of the section are t=8.1mm, T=13.3mm, D=454.6mm and B=152.9mm. Both its flanges and web are class 1 according to clause 5.5 of EN1993-1-1 (BSI 2005). The choice of this section size is driven by stiffness rather than resistance requirements.

The optimal section obtained for the customized beam has the following dimensions: t=5.4mm, T=8.0mm, D=561.9mm, B=187.2mm.

It is worth noting that, for the cases where web shear buckling is accounted, the results obtained are similar irrespective of whether web shear buckling is accounted for.

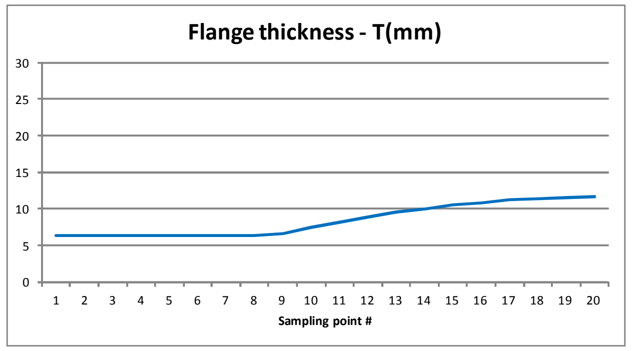
1 DIMENSION VARYING

t varying



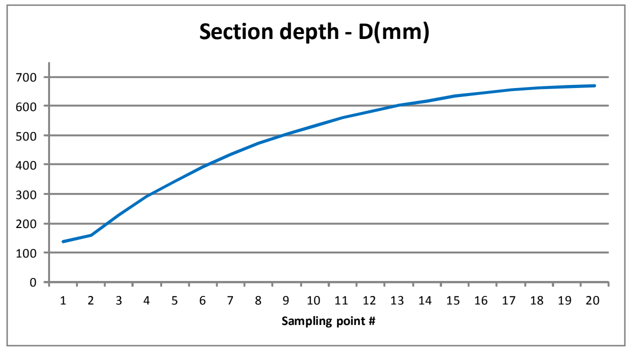
t	varies
T	7.5mm
D	555.3mm
B	175.7mm

T varying



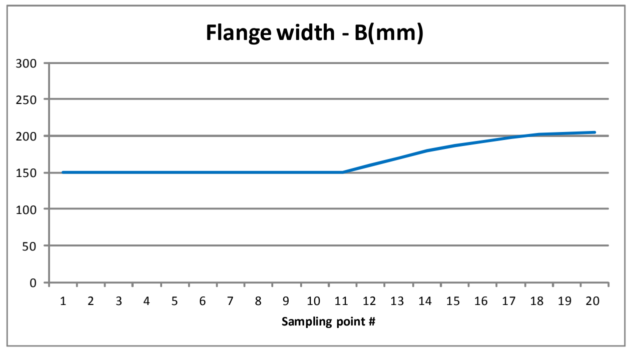
t	5.1mm
T	varies
D	523.8mm
B	150.0mm

D varying



t	6.5mm
T	6.3mm
D	varies
B	150.0mm

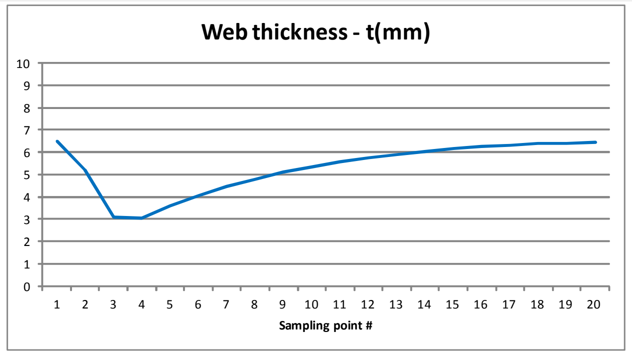
B varying

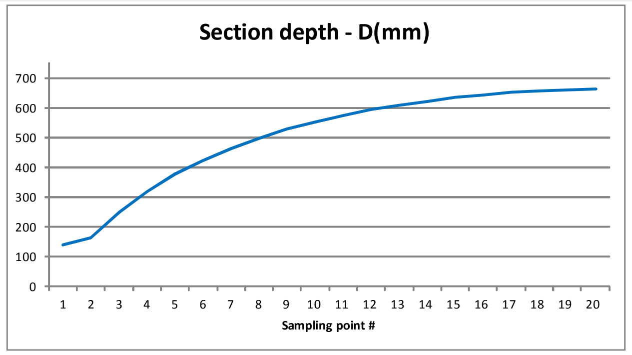


t	4.9mm
T	8.8mm
D	508.8mm
B	varies

2 DIMENSIONS VARYING

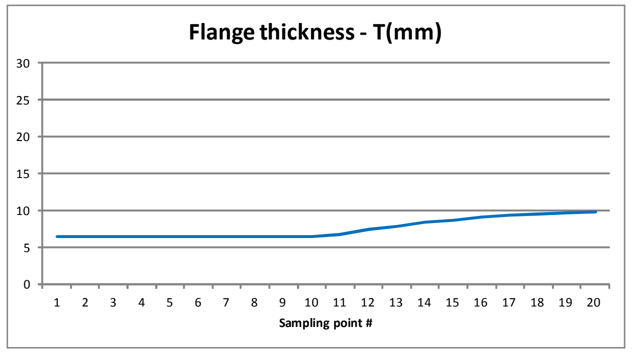
t & D varying

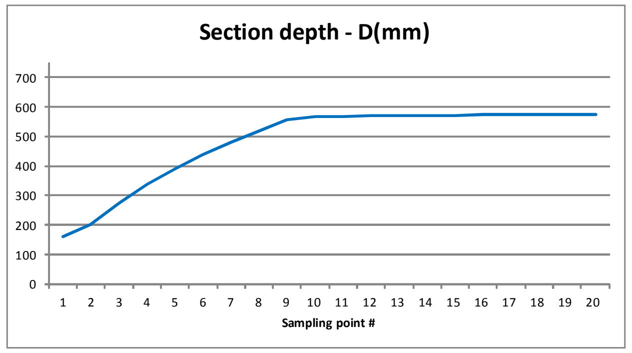




t	varies
T	6.5mm
D	varies
B	151.0mm

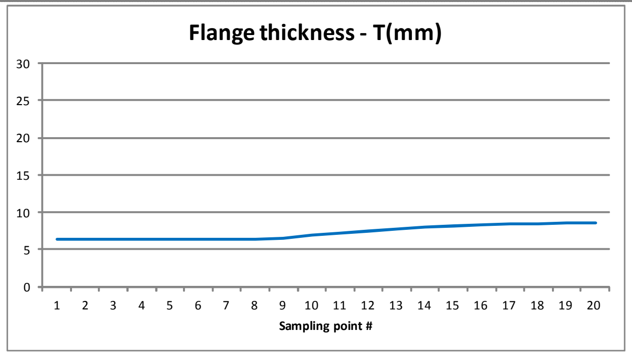
T & D varying

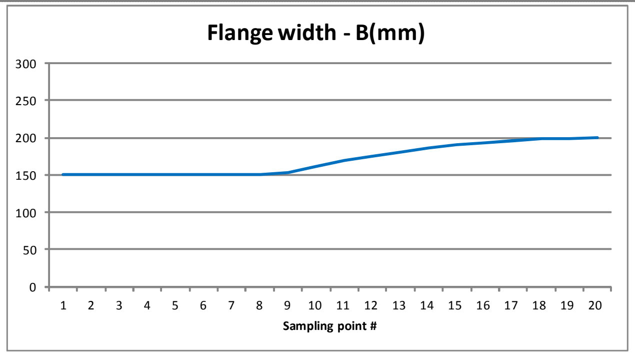




t	5.5mm
T	varies
D	varies
B	150.0mm

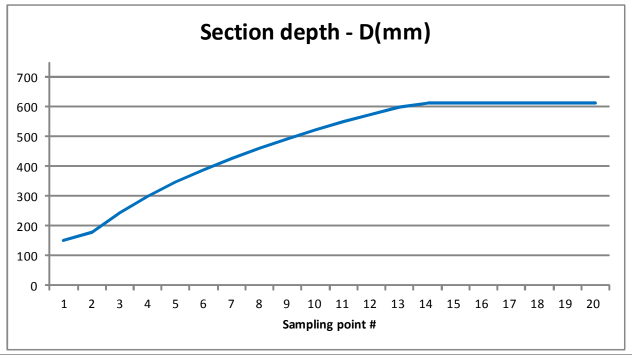
T & B varying

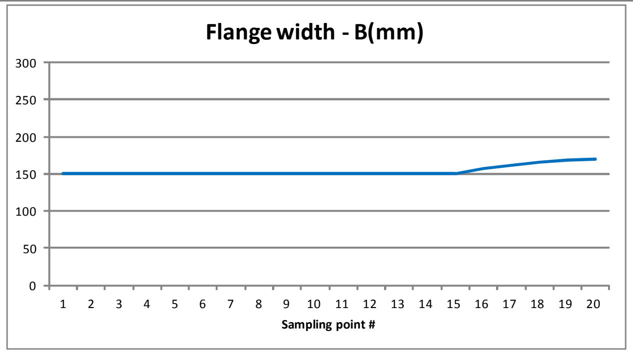




t	5.1mm
T	varies
D	523.8mm
B	varies

D & B varying





t	5.9mm
T	7.2mm
D	varies
B	varies

Figure 22a: Profiles of varying dimension(s) along half length of beam for analysis cases of Table 2 – Section depth unconstrained

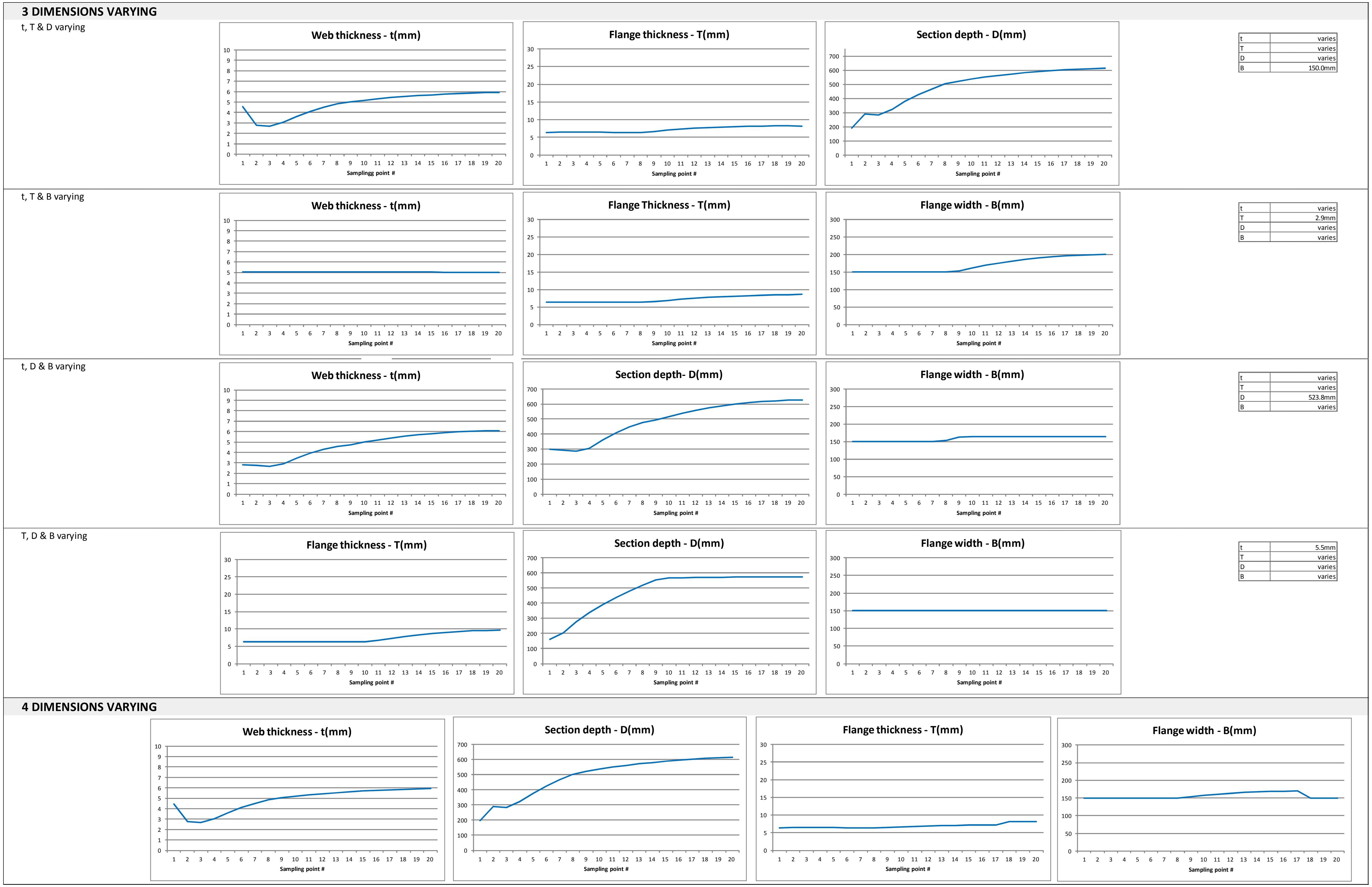


Figure 22b: Profiles of varying dimension(s) along half length of beam for analysis cases of Table 2 – Section depth unconstrained

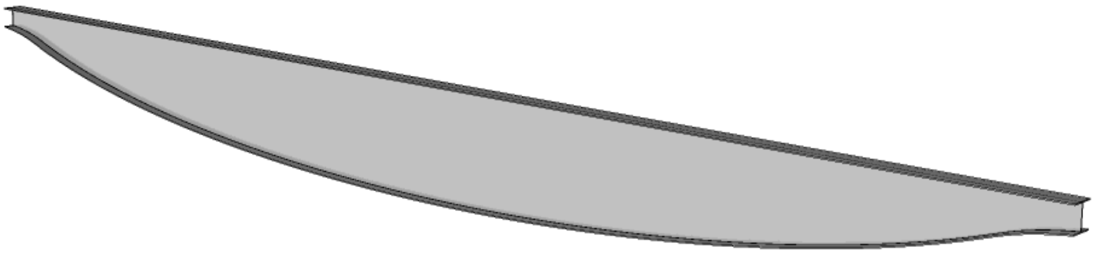


Figure 23: Profile of optimal beam with D varying – case where maximum depth is unconstrained

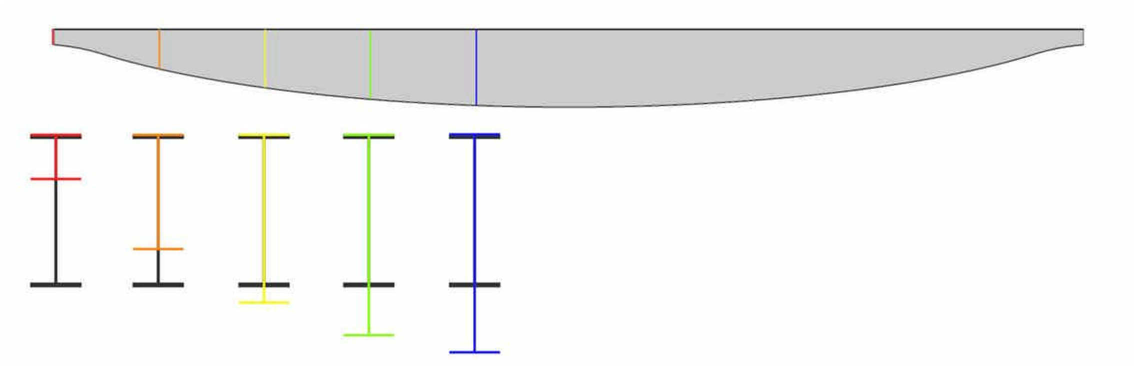


Figure 24: Elevation of and successive sections though optimal beam with varying depth – case where maximum depth is unconstrained. Section of catalogue beam used for comparison shown in grey.

3.2.2.2 Interpretation

This section attempts to interpret the profiles of the section dimensions obtained in each case and presented above.

3.2.2.2.1 Customised beam

For the section obtained for the customised beam, the constraints which are active are the bending resistance in segment 20 - at midspan where the applied bending moment is the greatest -, web and flange local buckling - at all segments as the section is constant along the beam's length.

Sections of class 3 only are considered in this investigation, so a section where the web and flanges are as slender as possible results in the strongest and stiffest section obtainable for a given material quantity: there are no benefits to be obtained from using stockier section elements as the increased bending resistance to be obtained from using a class 1 or 2 section is not considered. This justifies the result obtained.

3.2.2.2.2 Flange thickness T varying

In the case where the flange thickness *T* is allowed to vary, the active constraints are the midspan deflection under the quasi-permanent load combination, bending resistance in segments 9 to 20, and web and flange buckling in segments 1 to 8. The flange width *B* is also found to be set at its minimum allowed value of 150mm.

The profile obtained shows that the flange thickness is constant between segments 1 and 8 – which is consistent with the fact that flange buckling is active at these segments as the flange width cannot vary – and then varies between segments 9 and 20 following a shape close to a parabola – which is consistent with the fact that the bending resistance constraint is active at these segments and with the shape of the applied bending moment in a simply-supported beam, which follows a parabola.

It is worth noting that the fact that web buckling is not an active constraint in segments 9 to 20 is due to the increase in the flange thickness, which results in a reduction of the clear length between the bottom surface of the top flange and the top surface of the bottom flange. It is however very close to being active.

The fact that the flange thickness *T* does not vary along the whole length of the beam may be unexpected, as a variation along a greater length should result in additional weight savings. This partly finds its explanation in the fact that the flange width is set at its minimum value: *T* cannot continue to follow the parabolic profile obtained between segments 9 and 20 along the whole length of the beam, as the flange would otherwise not comply with the requirement for the section to be class 3. This is confirmed by the profile obtained if the lower bound imposed on the flange width *B* is removed. This profile is presented on Figure 25. In this case, the flange thickness varies through a greater portion of the length of the beam, between segments 5 and 20, and is again found to follow a parabola. However, it still does not vary along the whole length of the beam.

As explained above, a section in which the web and the flanges are as slender as possible results in the most efficient section. In the case considered here, *B* is not allowed to vary. The flanges are as slender as possible in the portion of the beam where *T* is constant, but it is not in the portion where it varies. A balance needs to be found between the savings to be obtained from allowing *T* to vary in a greater portion of the length and the loss of optimality of the flanges in the portion on which it varies. This explains the result obtained.

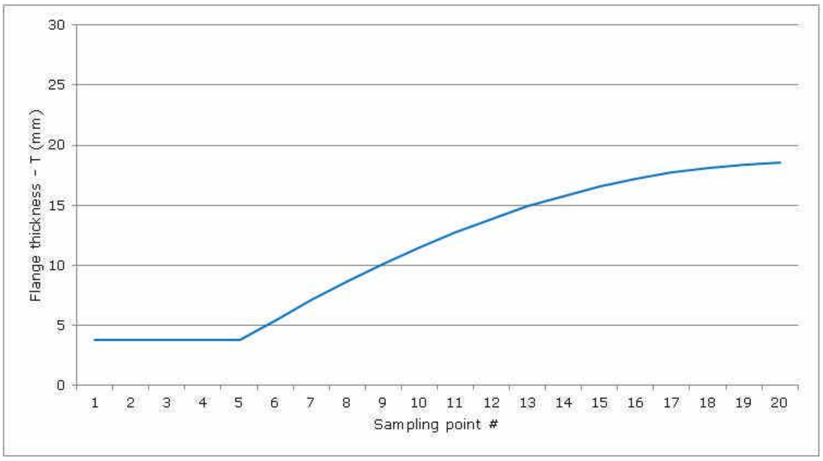


Figure 25: Variation in flange thickness *T* when *T* alone is allowed to vary along the beam's length – Case where lower bound constraint of 150mm imposed on flange width *B* is not considered

3.2.2.2.3 Flange thickness T and flange width B varying

The case where both of the flanges dimensions, *T* and *B*, vary closely relates to the previous case and is reviewed here.

In this case, the constraints found to be active are the midspan deflection under the quasi-permanent load combination, the bending resistance in segments 9 to 20, web buckling at sections 1 to 8, and flange buckling in all segments. The flange width is also found to be at its minimum allowed value of 150mm in segments 1 to 8.

The profiles obtained show that the two flange dimensions are constant in segments 1 to 8, and then vary between segments 9 and 20, following profiles close to parabolic. The fact that they are constant in a portion of the length of the beam is due to the lower bound constraint imposed on the flange width which is not allowed to be less than 150mm.

To verify this, the constraint imposing a lower limit on the flange width *B* is removed. The profiles obtained are presented in Figure 26 and Figure 27. Both dimensions are found to vary in a greater portion of the beam’s length but still do not vary on the whole length. A review of the active constraints in the obtained result provides the explanation for this. The active constraints are the midspan deflection under the quasi-permanent load combination, the bending resistance in segments 4 to 20, web buckling in segments 1 to 4, flange buckling in segments 5 to 20 and a geometric constraint. In segments 1 to 3, the constraint placed on the slenderness of the flange, ensuring that $B_i>5T_i$, becomes active. Both flange dimensions could be further reduced, but reducing *T* would increase the clear length between the bottom surface of the top flange and the top surface of the bottom flange thus requiring an increase in web thickness *t* along the whole length of the beam. This increase must result in a greater increase in weight than the reduction to be obtained by varying *B* and *T* in segments 1 to 3. This explains the profiles obtained.

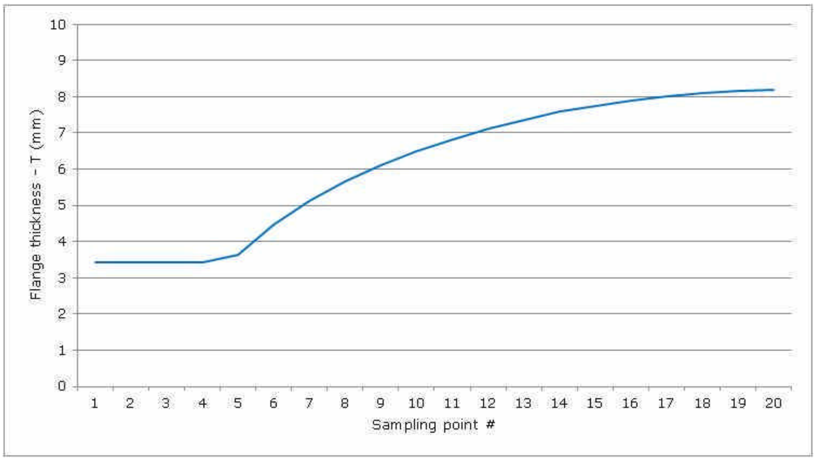


Figure 26: Variation in flange thickness *T* when *T* and *B* are allowed to vary along the beam’s length – Case where lower bound constraint of 150mm imposed on flange width *B* is not considered

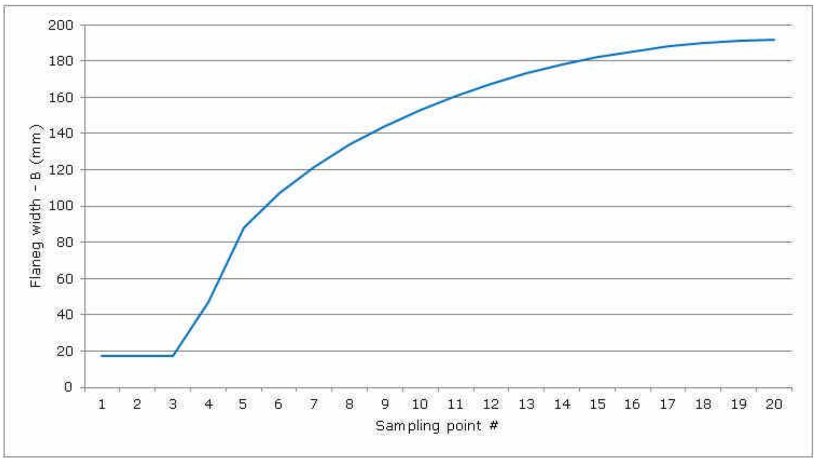


Figure 27: Variation in flange width *B* when *T* and *B* are allowed to vary along the beam’s length – Case where lower bound constraint of 150mm imposed on flange width *B* is not considered

3.2.2.2.4 Web depth D varying

In the case where the section depth is allowed to vary, the active constraints are the midspan deflection under the quasi-permanent load combination, the bending resistance in segments 2 to 20, the shear resistance in segment 1 – at the support -, web buckling in segment 20 – at midspan – and flange buckling in all segments.

The obtained profile of the depth is close to a parabola between segments 2 and 20 at which the bending resistance is an active constraint, and takes a different value at near the support in segment 1, which is dictated by the fact that the shear resistance becomes an active constraint.

An obvious way for this beam to be made more efficient is to allow the web thickness to vary, so that the web can be at its maximum allowed slenderness ratio along the whole length of the beam. This is discussed in the next section.

3.2.2.2.5 Web thickness t and section depth D varying

In the case where the two dimensions defining the web of the section are allowed to vary, the active constraints are the bending resistance in segments 4 to 20, the shear resistance in segments 1 to 3, web buckling in segments 4 to 20 and flange buckling in segment 4. For a given set of flange dimensions, flange buckling depends on the thickness of the web as this influences the length of the outstand flange. This influence is however marginal, and flange buckling is in fact very close to being active in all segments. It is also worth noting that the constraint related to the midspan deflection under the quasi-permanent load combination is also close to becoming active.

The profiles obtained for the web dimensions are in accordance with the situation forecast in the previous section: the section depth again follows a profile close to parabolic. The web thickness evolves in a similar manner and brings the web at its largest allowed slenderness ratio through most of the length of the beam.

3.2.2.2.6 Flange width B varying

In the case where the flange width is allowed to vary, the active constraints are the midspan deflection under the quasi-permanent load combination, the bending resistance in segments 12 to 20, web buckling in all segments, and flange buckling in segment 20. Additionally, the flange width is found to be at its minimum allowed value of 150mm in segments 1 to 11.

The profile obtained shows that the flange width is constant between segments 1 and 11, and then follows a profile close to parabolic between segments 12 and 20. This profile is similar to that obtained in the case where the flange thickness T varies.

A variation of the flange width in a longer portion of the beam may have resulted in a lighter solution. Using thinner flanges would have required a wider flange and may hence have resulted in a variation in the flange width on a greater length. This is however made impossible by the fact that flange buckling is active in segment 20.

Another valid question is whether a shallower section would have resulted in a lighter beam. Indeed, a section with a reduced lever arm between the flanges would require more material to be placed in the flange. This may result in a variation in flange width in a larger portion of the beam's length which may compensate for the fact that the section is shallower. To test this scenario, the algorithm is re-run with an upper-bound of 400mm placed on the section depth.

The profile obtained for the flange width is presented in Figure 28. The flange width is indeed found to start varying earlier than when the section depth is unconstrained. Another difference with the profile previously obtained is that the flange remains constant in segments 15 to 20. The active constraints in this case are the midspan deflection under the quasi-permanent load combination, web buckling in all sections and flange buckling in sections 15 to 20. The flange width is also at its minimum allowed value in segments 1 to 4. The reason why the flange width does not keep on varying in segments 15 to 20 is that this would require thicker flanges for the sections in these segments to comply with local buckling requirements. This increase would penalise the sections in all the other segments as the flange thickness is constant along the beam's length. This is confirmed by Figure 29 which shows the profile of the flange width obtained when the section depth is still limited to 400mm, but the flange thickness is allowed to vary – the profile of the flange thickness is shown in Figure 30. In this case, the flange width varies between segments 5 and 20.

Considering the weights obtained when the flange width is allowed to vary, when the section depth is unconstrained, the optimal beam obtained weighs 377.1kg compared to 472.3kg when it is constrained to 400mm. Even when the two dimensions defining the flanges are allowed to vary, when the section depth is constrained to 400mm, the weight of the beam obtained is 448.0kg. This answers the initial question and shows that a variation in flange width in a greater proportion of the beam to be obtained from using a shallower section does not counterbalance the resulting loss in structural-efficiency.

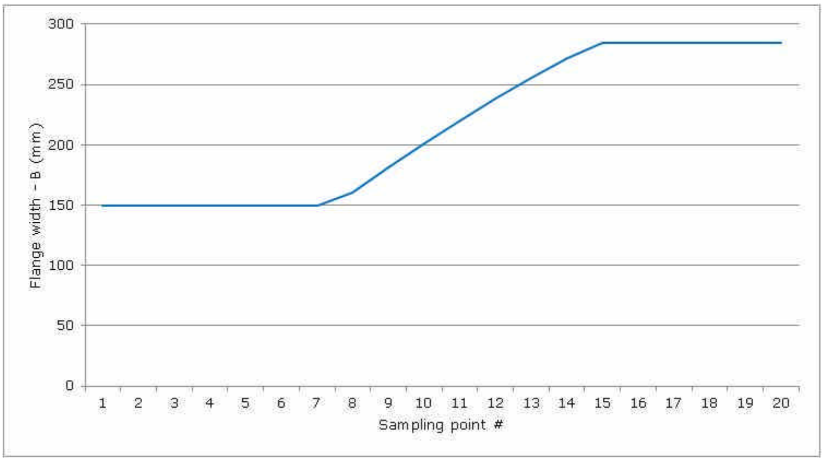


Figure 28: Variation in flange width *B* when *B* alone is allowed to vary along the beam's length – Case where additional upper bound constraint of 400mm is imposed on section depth *D*

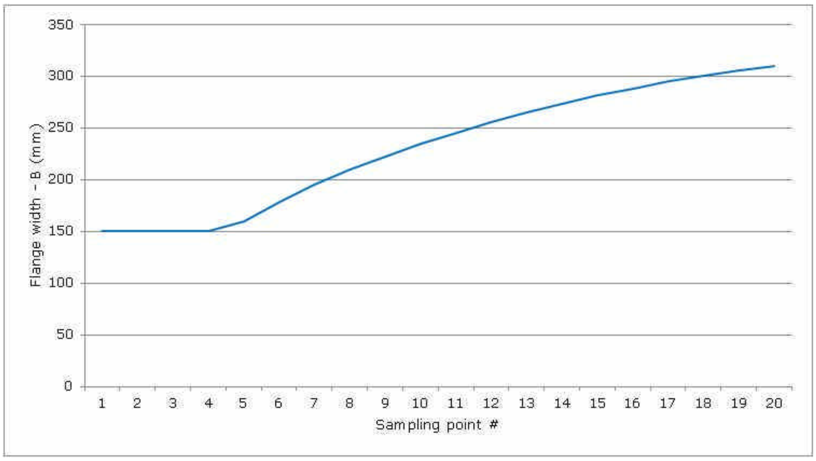


Figure 29: Variation in flange width *B* when *B* and *T* are allowed to vary along the beam's length – Case where additional upper bound constraint of 400mm is imposed on section depth *D*

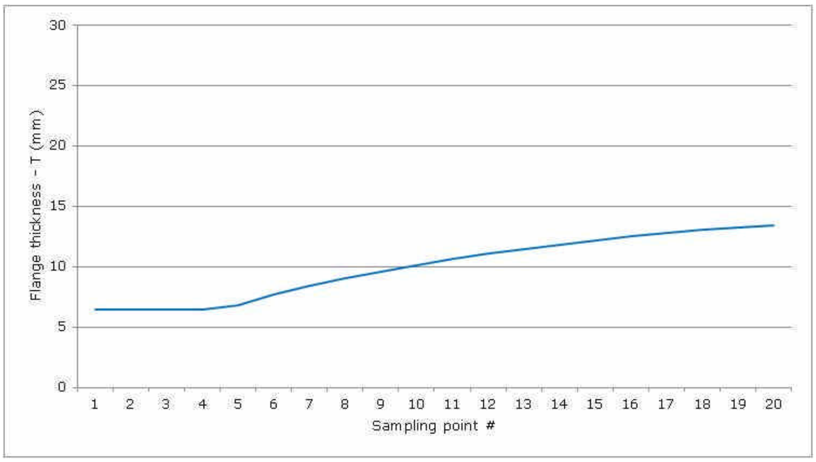


Figure 30: Variation in flange thickness *T* when *B* and *T* are allowed to vary along the beam's length – Case where additional upper bound constraint of 400mm is imposed on section depth *D*

3.2.2.2.7 Flange thickness T and section depth D varying

In the case where the flange thickness and the section depth vary along the beam's length, the active constraints are the midspan deflection under the quasi-permanent load combination, the bending resistance in segments 11 to 22, the shear resistance in segment 1 – at the support -, web buckling in segments 10 to 20, and flange buckling in segments 1 to 10. The flange width, which is kept constant, is set to its minimum allowed value.

Regarding the profiles obtained, the flange thickness is constant in segments 1 to 10 and varies following a profile close to parabolic in segments 11 to 20. The section depth is the opposite: it is constant in segments 11 to 20 and varies between segments 1 and 10.

The reason why these profiles are obtained comes from the fact that a balance is found between the need to satisfy local buckling requirements while keeping the beam's weight to a minimum: a variation in the section depth along the whole beam's length, as obtained in Figure 22 when the section depth alone is allowed to vary, would require a thicker web for local buckling. This would penalise all the segments using a shallower section depth which do not require such a thick web. Instead, in the solution obtained, the section depth is kept constant in a portion of the beam, which keeps the web reasonably thin, and the flange thickness starts varying. The reasoning is effectively the same as far as the flange thickness is concerned.

3.2.2.2.8 Flange width B and section depth D varying

The profiles obtained for the case where the flange width and the section depth are allowed to vary are very similar to those obtained when the flange thickness and the section depth vary. The reasons behind the obtained profiles are also similar.

3.2.2.2.9 Cases where more than two dimensions are allowed to vary concurrently

The profiles obtained for the cases where more than two dimensions are allowed to vary concurrently are given in Figure 22b. Since the number of active constraints increases with the number of dimensions varying, giving a similar interpretation of the results as is given for the cases reviewed above becomes complicated. The profiles obtained however follow principles similar to those described for the eight cases reviewed above. The lists of active constraints are given in Appendix E should the reader want to get a deeper understanding of the results.

3.2.3 Weights of beams

Figure 31 presents the weights obtained for the beams corresponding to the 13 analysis cases listed in Table 2 when the section depth *D* is unconstrained. It also shows the obtained percentage reductions in weight compared to the lightest viable constant catalogue section beam. As explained in Section 2.2.3.3, for the cases where the web dimensions are constant, web shear buckling has been accounted for. These cases are indicated with a * above the dimension(s) varying on the horizontal axis of Figure 31.

Figure 32 summarises the information of Figure 31: it gives the maximum weight savings achievable as a function of the number of dimensions varying. This figure aggregates data relative to cases where web shear buckling has been accounted for together with cases where it has not. It is therefore not appropriate for construction but nevertheless yields interesting results for this comparative study.

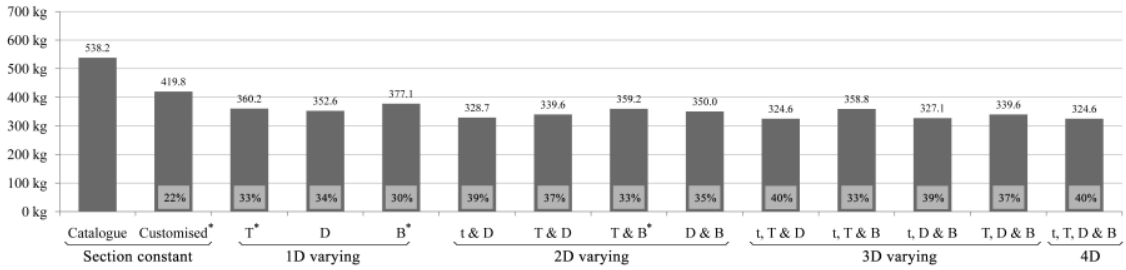


Figure 31: Beam weights for the 13 analysis cases in Table 1 – unconstrained section depth (* indicates the cases for which web shear buckling is accounted for)

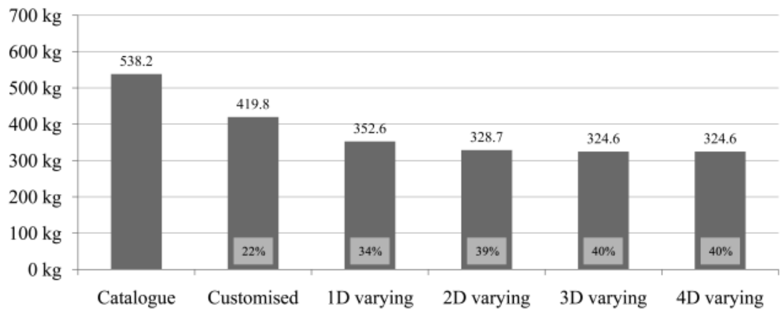


Figure 32: Minimum beam weights as function of number of dimensions varying – unconstrained section depth

3.2.4 Discussion of unconstrained case

The results indicate that bespoke steel I-beams with varying section have the potential to be much lighter than constant catalogue section beams. In the particular case under consideration, a reduction in steel weight of at least 33% can be obtained. The result obtained for the case where all four dimensions are allowed to vary is not a definite result as web shear buckling is not accounted for, but indicates that the maximum savings achievable would lie below 40%. A post-study using FEM would need to be carried out to confirm a precise figure.

A large proportion of these savings comes from the fact that there is no catalogue section which closely matches the requirements of the particular design configuration considered. Indeed, using a constant customised section beam instead of a catalogue one yields a reduction of 22% in steel weight. The constant catalogue section indeed does not match the design configuration considered: As highlighted in Section 0, both its flanges and web are class 1 while the choice of this section size is driven by stiffness rather than resistance requirements. In this design situation, there is no benefit for the section to be class 1 and slenderer flanges and webs would have resulted in a stiffer section for the same material quantities. The obtained weight saving is obviously particular to the design configuration used for the present investigation and cannot simply be generalised. It is however indicative of the potential benefits of using bespoke constant section beams instead of working with a standardised finite set of section sizes such as that offered by steel manufacturers catalogues.

Overall, the results follow the law of diminishing returns, as made obvious in Figure 32. As complexity – number of dimensions varying – increases, the additional savings obtainable decrease to finally reach a plateau. The improvements obtainable by varying more than two dimensions concurrently are in fact only marginal and are unlikely to justify the corresponding increase in the complexity of the required rolling mill.

Most of the savings obtained by allowing the section dimensions to vary come from a variation in only one of the four dimensions at a time. A third of the weight of the constant catalogue section beam is saved by varying the flange thickness. This corresponds to a 16% reduction over the weight of the constant customised section beam. Varying the section depth would, at best, result in a weight saving of a similar order. Further detailed study into the influence of web buckling would be required to confirm this result. Variations in the flange width yield slightly less savings. This is likely due to the lower limit placed on the flange width to ensure appropriate bearing area for the above slab, as this results in a variation in the flange width in a shorter portion of the beam’s length than when the flange thickness is allowed to vary.

Simultaneous variations in two dimensions may push the savings further. The maximum savings which can be expected would be 39% of the weight of the constant catalogue section beam, or 21% of the weight of the constant customised section beam. The improvement over the cases where one dimension is allowed to vary are however limited and would need to be assessed against the implications on the complexity of the rolling mill.

3.3 Results for constrained section depth

In the set of results presented in the previous section, the catalogue section, a UKB 457x152x60, is 454.6mm deep, a depth exceeded by the optimal beams obtained for most of the analysis cases. In the case where the section depth *D* alone is allowed to vary for example, *D* reaches 670mm at midspan.

To take into account the fact that in multi-storey office building design, a heavier shallower beam is likely to be preferred to a lighter but deeper one, the same study as presented above is repeated with two different upper bounds placed on the section depth.

3.3.1 Input parameters

The values of these limits correspond to the depths of respectively the lightest viable catalogue section beam and the shallowest viable catalogue section beam for the design configuration considered.

The lightest viable catalogue section beam is, as before, a UKB 457x152x60 (Tata Steel 2011). In the first case, the section depth is hence constrained to the depth of this section, at 454.6mm.

The shallowest viable catalogue section beam is a UKB 406x178x74 (Tata Steel 2011). In the second case, the section depth is hence constrained to the depth of this section, at 412.8mm. As previously, the choice of this section is not driven by resistance considerations, but by stiffness. It should be noted that with a linear weight of 74.2kg/m, this beams weighs significantly more than the lightest viable one which weighs 59.8kg/m.

Again, the results presented in this section have been obtained by discretising the beam in 20 segments, i.e. *N=20*.

3.3.2 Profiles of beams

As for the first set of results, the profile obtained for the different analysis cases listed in Table 2 are presented.

3.3.2.1 Results

The profiles of the dimensions varying along the length of the beam are presented in Figure 33 and Figure 34 for the cases where the section depth is constrained to 454.6mm and 412.8mm respectively. The exact dimensions are also given in a tabular form in Appendix C and D respectively.

Figure 35 and Figure 36 respectively present a 3D view and an elevation of the beam obtained when the section depth is allowed to vary along the length, and is constrained to 454.6mm.

In the case where the section depth is constrained to 454.4mm, the optimal section obtained for the customized beam has the following dimensions: t=4.3mm, T=9.7mm, D=454.6mm, B=226.0mm.

In the case where the section depth is constrained to 412.8mm, the optimal section obtained for the customized beam has the following dimensions: t=3.9mm, T=11.0mm, D=412.8mm, B=255.6mm.

In the set of results where the section depth is unconstrained, the customised section beam uses a section depth of 561.9mm, which is larger than the two upper-bounds considered here. As expected, the section depth of the customised section beams in the two cases where the depth is constrained are found to be at their maximum allowed value.

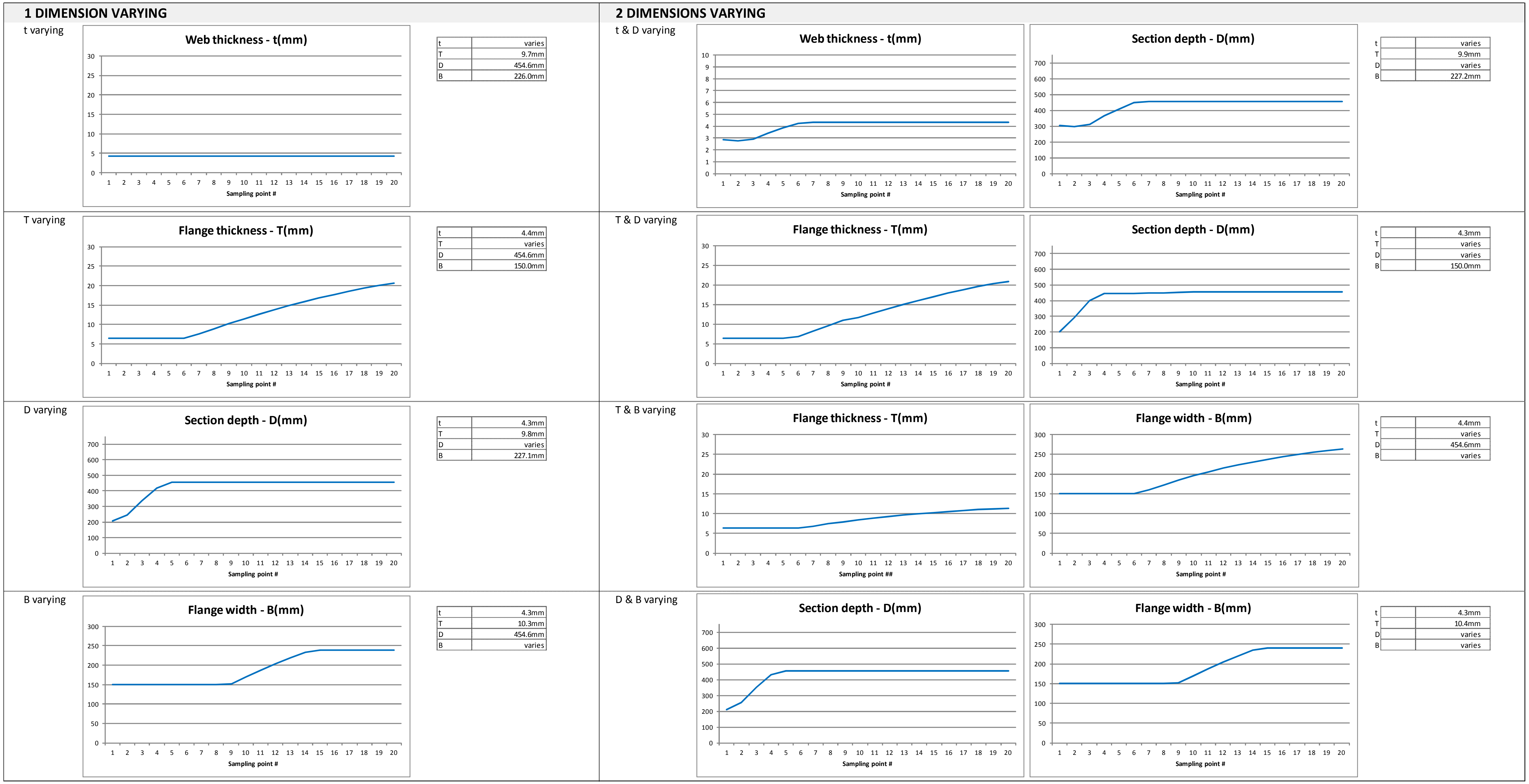
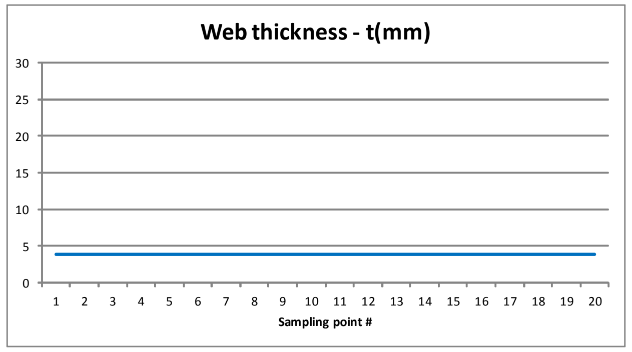


Figure 33: Profiles of varying dimension(s) along half length of beam for analysis cases of Table 2 – Section depth constrained to 456.4mm

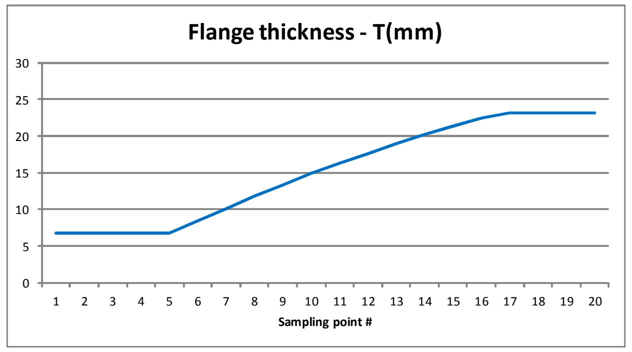
1 DIMENSION VARYING

t varying



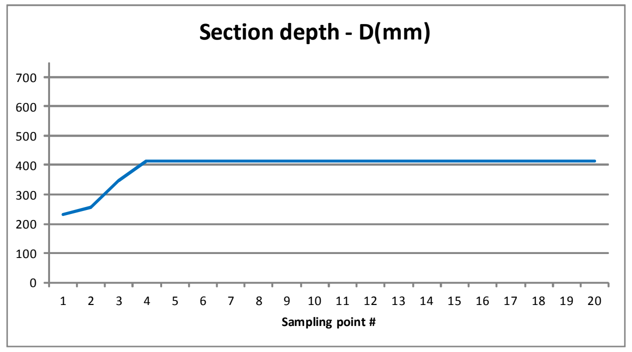
t	varies
T	11.0mm
D	412.8mm
B	255.6mm

T varying



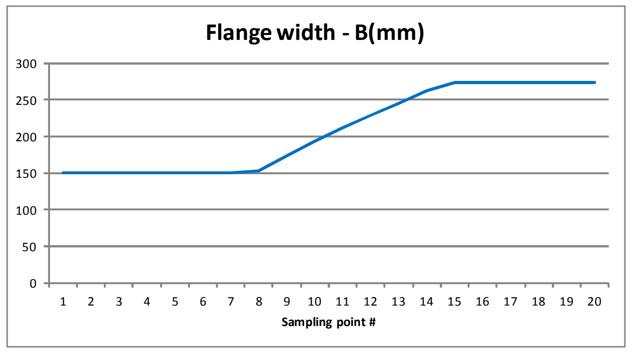
t	4.0mm
T	varies
D	412.8mm
B	158.8mm

D varying



t	3.9mm
T	11.1mm
D	varies
B	256.2mm

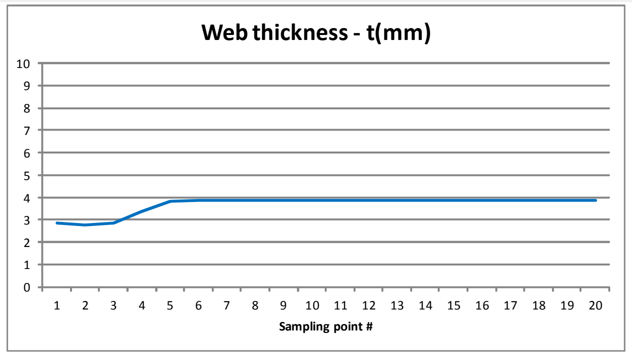
B varying

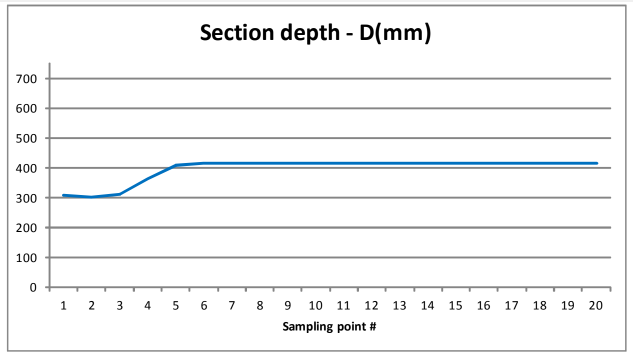


t	3.9mm
T	11.8mm
D	412.8mm
B	varies

2 DIMENSIONS VARYING

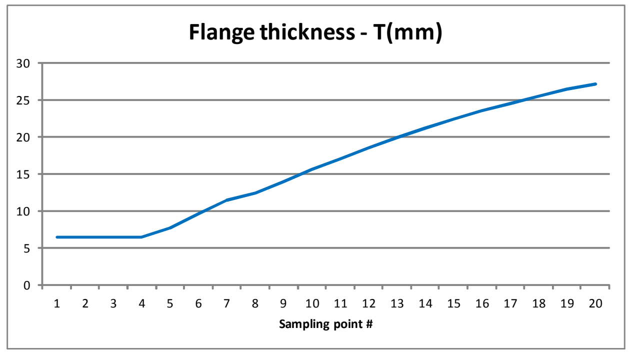
t & D varying

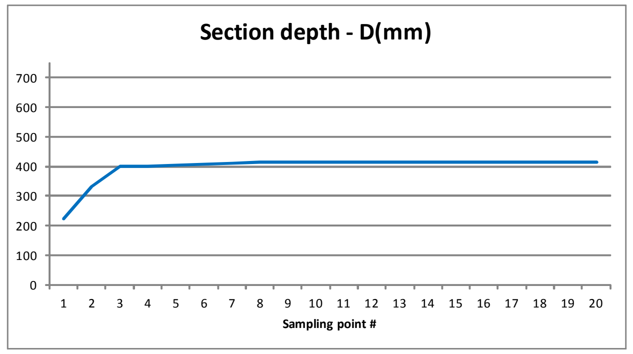




t	varies
T	11.1mm
D	varies
B	256.1mm

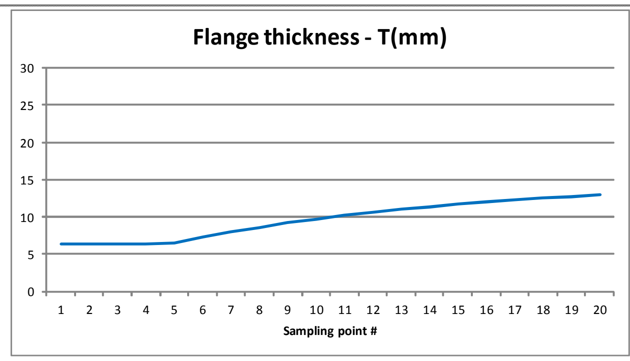
T & D varying

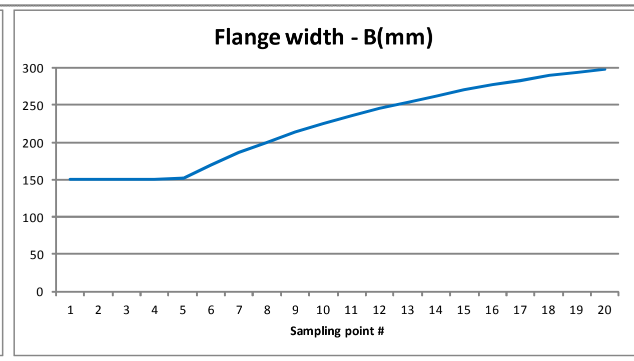




t	3.8mm
T	varies
D	varies
B	150.2mm

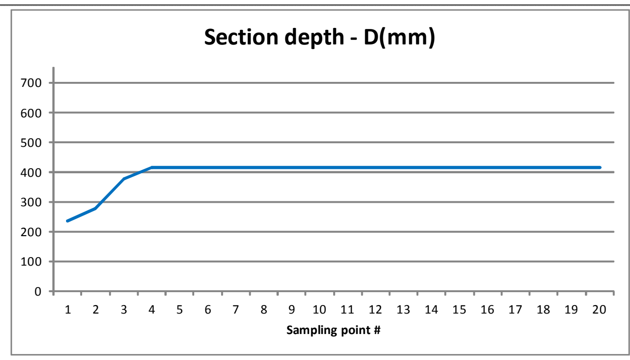
T & B varying

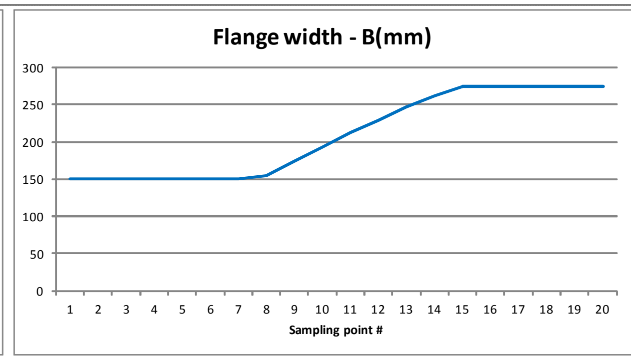




t	4.0mm
T	varies
D	412.8mm
B	varies

D & B varying





t	3.9mm
T	11.9mm
D	varies
B	varies

Figure 34: Profiles of varying dimension(s) along half length of beam for analysis cases of Table 2 – Section depth constrained to 412.8mm

It is worth noting that, for the cases where web shear buckling is accounted, the results obtained are similar irrespective of whether shear web buckling is accounted for, apart from the case where the flange thickness *T* is allowed to vary when the section depth is constrained to 412.8mm as made obvious in Table D 2 and Table D 3. The difference is weight between the two obtained beams is very limited: when shear web buckling is not accounted for, the optimal beam weighs 444.7kg, compared to 445.5kg when it is accounted for.

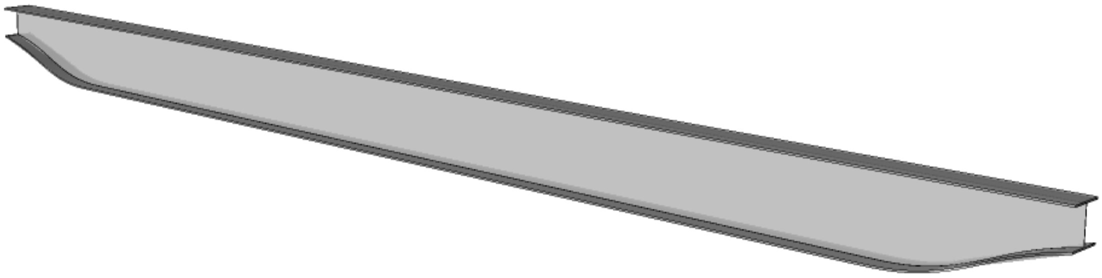


Figure 35: Profile of optimal beam with *D* varying – case where maximum depth is constrained to 454.6mm

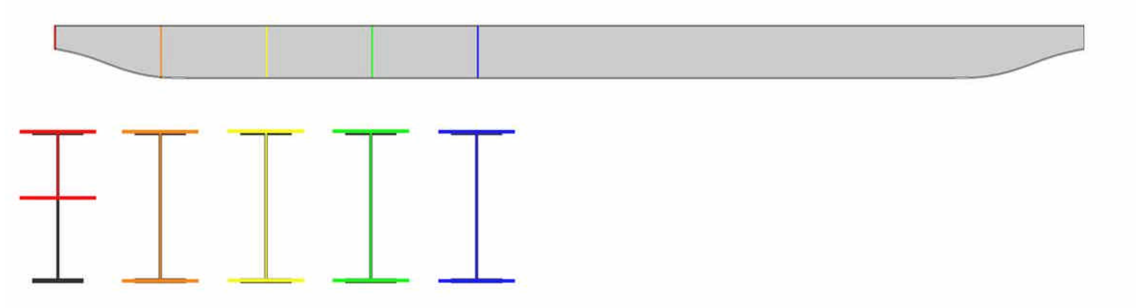


Figure 36: Elevation of and successive sections though optimal beam with varying depth – case where maximum depth is constrained to 454.6mm. Section of catalogue beam used for comparison shown in grey.

3.3.2.2 Interpretation

For most cases, the dimensions allowed to vary follow similar profiles as obtained previously in the case where the section depth is unconstrained. The main noticeable differences concern the cases where the section depth is allowed to vary along the length of the beam.

3.3.2.2.1 Section depth D varying

In the case where the section depth is constrained to 454.6mm when the section depth *D* alone is allowed to vary the profile obtained is similar to that obtained in the case where the section depth is unconstrained between segments 1 and 5. But at segment 5 the depth reaches its maximum allowed value and then remains constant through to segment 20.

In the case where the section depth is constrained to 412.8mm, the section depth varies in an even smaller portion of the length, as it reaches its maximum value at segment 4.

Since when the flange thickness or the flange width are allowed to vary, they vary in a similar portion of the length of the beam irrespective of whether the section depth is constrained, the merits of allowing the section depth to vary in cases where it is constrained are likely to be much less than that of allowing the flange dimensions to vary.

3.3.2.2.2 Web thickness t and section depth D varying

A similar situation as described above is obtained in the case where both the web thickness and the section depth are allowed to vary: although the section depth varies in a slightly longer portion of the length, both dimensions quickly reach their maximum allowed value.

3.3.3 Weights of beams

The weights obtained for the optimal beams are summarised in Figure 37 and Figure 38 for the cases where the section depth is constrained to 454.6mm and 412.8mm respectively. As previously, for each case, the obtained percentage reduction in weight compared to the lightest viable constant catalogue section beam is given in each bar. Also, the cases for which web shear buckling has been accounted for are indicated with an asterisk above the dimension(s) varying on the horizontal axis.

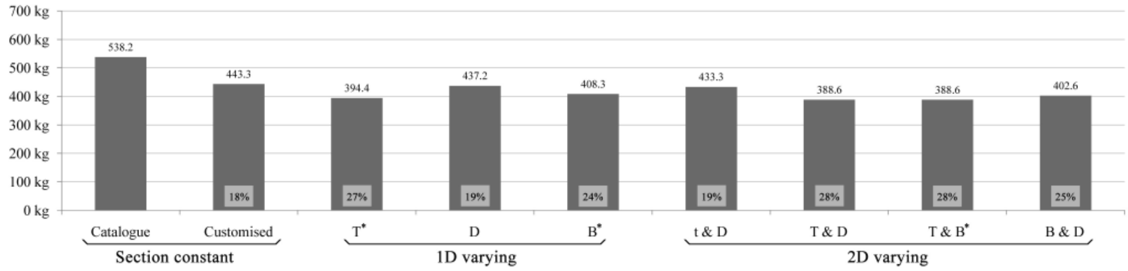


Figure 37: Beam weights for the 8 first analysis cases in Table 1 – maximum section depth constrained to 454.6mm (* indicates the cases for which web shear buckling is accounted for)

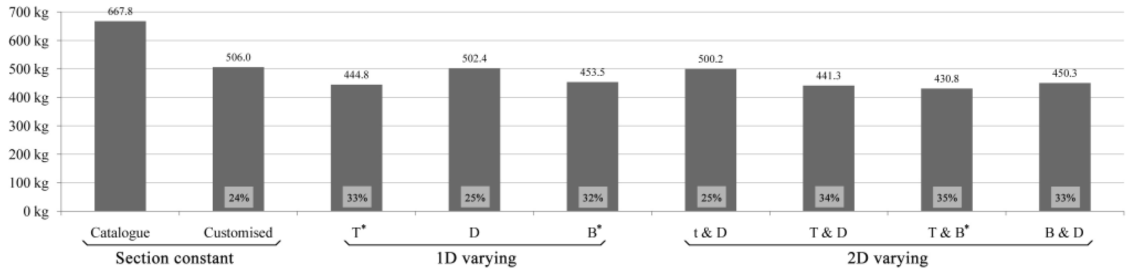


Figure 38: Beam weights for the 8 first analysis cases in Table 1 – maximum section depth constrained to 412.8mm (* indicates the cases for which web shear buckling is accounted for)

3.3.4 Discussion of constrained case

The results follow a pattern similar to that seen in the unconstrained case. Here however, the additional weight reductions obtained by varying more than one dimension at a time are clearly negligible.

Also, as in the unconstrained case, significant savings are obtained by using a constant customised section beam instead of a catalogue one. This indicates that the savings obtained when the section depth *D* is unconstrained are not merely due to the adoption of a section depth larger than that of the catalogue section used for comparison.

The main difference between the constrained and unconstrained cases concerns the relative merits of allowing each of the section dimensions to vary independently: unlike in the previous case, varying the section depth *D* is found to only produce minor weight savings. This is due, as explained previously, to the fact that *D* quickly reaches its maximum allowed value and can hence only vary in a limited portion of the length of the beam. Variations in either of the dimensions of the flanges are found to produce savings of an order similar to that obtained from a variation in the flange thickness in the unconstrained case, with a reduction of around 12% over the weight of the constant customised section beam, with variations in the flange thickness again found to result in greater savings than the width.

3.4 DISCUSSION

In this section, four aspects are discussed which respectively concern possible areas for further work, the potential for such a technology to contribute to other aspects of building design than reductions in structural weight, through greater integration between structure and services in particular, the need to develop an appropriate manufacturing process and the characteristics which such a process should be sought to display, and finally the challenges which the introduction of such a technology would represent for the steel section supply chain.

3.4.1 Further work

Possible areas for further work are first discussed here.

3.4.1.1 Assessing the potential merits of using customised constant section beams

The study presented points out that large weight savings may be obtained through the use of customised constant section beams instead of beams using standard catalogue sections available from steel manufacturers. This result is particular to the design configuration used for this investigation. To assess its generality, the potential savings to be obtained from using customised constant section beams would need to be assessed on a broader range of actual design situations. This could be done by considering several existing buildings in which steel beams are used, and testing the savings to be obtained.

Such a study would yield an understanding of whether the range of standard section sizes currently available is well-suited for the design situations encountered in the design of contemporary steel structures. It may highlight the need to reassess the range of standard section sizes. This may not require a complete change in the range of sections currently proposed: introducing a few more section sizes in certain ranges of sections may be found to cover most of the potential savings.

3.4.1.2 Addressing the limitations of the present study

The present study has a number of limitations as it only considers a limited range of section types, and leaves several structural aspects unaddressed. These limitations are discussed here.

3.4.1.2.1 Accounting for other section classes

As explained, in the study presented, only class 3 sections, which can develop their full elastic capacity, but cannot reach their plastic moment resistance, are considered. In several of the analysis cases considered, bending resistance is found to be an active constraint in certain segments of the optimal beams obtained. This suggests that allowing the beams' sections to become class 1 or 2 may result in greater weight savings than obtained. This may be implemented in the calculation of the bending moment resistance of the beam.

Likewise, allowing sections of the beam to become class 4 may result in greater savings than obtained. Additionally, it may also result in a different repartition of the relative merits of allowing different subsets of the section dimensions to vary. As explained in Section 2.2.3.2, this would require the use of FEM, as recommended by EN1993-1-5 (BSI 2005b, clause 2.5). This would be a significantly more involved undertaking than accounting for class 1 or 2 sections: the FEM modelling of a beam presents a challenge in itself, in particular regarding the modelling of its initial imperfections, as these influence the buckling behaviour of the element (Da Silva Vellasco 2001). Linking an optimisation algorithm to an FEM model would represent an additional challenge, not only in terms of implementation, but also because the calculation times required may become prohibitively long. Before going too far in this direction, an understanding of the potential merits of using class 4 sections would need to be obtained.

3.4.1.2.2 Accounting for web shear buckling

Accounting for the web shear buckling resistance of the web for the beams in which the web dimensions vary is another aspect which the present study leaves unaddressed. As explained in Section 2.2.3.3, EN1993-1-5 (BSI 2005b, clause 2.5) covers such cases through the use of FEM. The challenges highlighted in the previous section to account for class 4 sections would have to be addressed. Again, an understanding of the influence of this effect should be sought to be obtained before going too far in this direction.

3.4.1.3 Parameterisation

As explained in Section 2.3.1, the parameterisation of the beams chosen for this investigation aims at avoiding over-constraining the range of possible solutions. This is done as, at this stage, the choice of an actual fabrication method is open. As this progresses and a choice is made, the constraints of the method could be included in the parameterisation of the beam: this may include the addition of a limit on the allowable rate of change in a certain dimension of the beam section along the length, or the use of a family of shape functions to describe the capabilities of the rolling mill.

3.4.1.4 Investigating other geometries

As explained in Section 2.2.1, the range of geometries investigated for the beams is in accordance with the capabilities foreseen by the rolling mill manufacturer for the mill. As such, the beams studied use an I-section and the thickness of the flanges and web are constant in each section. Opening the range of possibility may result in even lighter elements and may be worth considering. In particular, if web shear buckling was found to be an issue in the cases where the web dimensions are allowed to vary, the introduction of web stiffeners by rolling thicker localized areas along the length of the web may be considered. Again, this would have to be linked to the capabilities of the developed rolling mill.

3.4.2 Potential for greater integration

The study and conclusions presented here are limited to potential savings in steel weight. Structural weight is however only one aspect of building design, and not necessarily the most important. In the design of multi-storey office buildings in particular, adequate integration of structure and services is in practice likely to be given priority over structural weight aspects. Indeed the impact which such integration may have on the overall building design, and one can assume, on its overall embodied environmental impact – through reductions in floor-to-ceiling height resulting in a decreased façade area and height of internal walls and partitions etc. – could be greater than the impact of reductions in structural weight. The main benefit of the introduction of such a technology may lie not so much in the potential weight savings achievable as in the greater flexibility it would offer designers to produce more integrated designs. For this reason, a rolling mill enabling variations in section depth may still be considered for the multi-storey office market.

3.4.3 Manufacturing processes

The fact that the element shapes currently used in buildings are not structurally optimal is a well-known fact. The main issue is the absence of an adequate manufacturing process which would make it possible to produce such shapes in a commercially viable manner and result an overall material saving. Carruth et al. (2012) are investigating the potential to manufacture beams with a variable depth by hot-rolling. As explained by the authors, hot-rolling has the preference as it results in minimal yield losses. Existing manufacturing techniques effectively already enable the fabrication of beams in which all four section dimensions can vary: plates of varying thicknesses can be rolled (Urban 2006) which can then be cut to any desired profile and welded together. Such beams may however not result in overall material savings as subjected to high scrap rates. Moreover, they do not compete on economic grounds due to the extra steps involved in their manufacturing.

3.4.4 Supply chain integration

If the success of such a technology is largely dependent on the development of an appropriate manufacturing technique, its real-life deployment is likely to prove equally challenging for the steel sections supply chain, irrespective of the shape of the beams. Indeed, working with bespoke beams, even customised constant section ones which, from a pure technical point of view, could be hot-rolled today, would require a change to a just-in-time paradigm, each beam having to be manufactured to order. Achieving this would imply moving from the current half predictable pattern of order, where only a finite number of sizes are used and can be stocked, towards a model where greater flexibility and responsiveness are central. The reassessment of the range of catalogue sections offered by manufacturers mentioned in Section 3.4.1.1 may be an easier measure to implement until fabrication to order is made feasible.

3.5 CONCLUSION

The study presented in these two chapters highlights that using bespoke steel I-beams with varying section has the potential to yield significant weight savings over currently standard constant catalogue section beams. For the design configuration used for investigation, which is largely representative of contemporary office building design, a weight reduction of between 33% and 40% has been shown possible. This result exemplifies the potential merits of using more material-efficient structural element shapes in reducing the embodied environmental impact of building structures. Eurocode 3 accounts for local buckling through section classification. The present study only considers class 3 sections and further savings than demonstrated here may be possible by allowing the section to become class 1, 2 or 4 along the length of the beam. As discussed, further work is needed to fully assess these potential further savings.

The merits of allowing different sets of a beam section dimensions to vary are investigated for two cases: one with the section depth unconstrained and one where it is constrained in the upper range to reflect the fact that in multi-storey office buildings, floor-to-ceiling height, and hence depth of the structural zone, is often sought to be minimised.

In both cases, a significant proportion of the total potential savings is obtained by using a bespoke constant section beam, the dimensions of which do not vary but are optimally chosen to produce a minimum weight design. This result, which points out the disadvantage associated with standardisation, is obviously particular to the design configuration used and cannot be easily generalised. It is however indicative of the possibilities; further work is needed to test this result against a series of different design configurations in order to fully assess the merits of such beams.

Variations in the section dimensions yield further savings and follow the law of diminishing returns, with variations in more than two dimensions only producing marginal additional savings.

With the section depth unconstrained, when one dimension is allowed to vary, varying the flange thickness results in a 33% saving compared to the lightest catalogue section beam available which corresponds to a 16% reduction over the weight of the constant customised section beam. Varying the section depth would at best result in a weight saving of a similar order. A definite answer on the savings achievable in this case can however not be given here as web shear buckling was not accounted for: it would require more detailed FEM analysis which is beyond the scope of this study. Varying two dimensions concurrently may push the savings further. The maximum savings which can be expected would be of 21% of the weight of the constant customised section beam. The improvements over the cases where one dimension is allowed to vary are limited and would need to be assessed against the implications on the complexity of the rolling mill.

In the case where the section depth is constrained, the improvements obtained by varying more than one dimension are negligible. Variations in either of the flange dimensions yield the greatest savings while varying the section depth barely makes an improvement over the customised constant section beam.

The UK consumption of steel sections is mainly shared between two building typologies: single storey industrial buildings and multi-storey office buildings (BCSA 2010). Each is representative of one of the design situations highlighted above: respectively maximum section depth unconstrained and constrained. Assuming that the present results hold true for the former typology, a rolling mill enabling variations in the flange thickness would deliver most of the potential savings while adequately covering both sectors. Varying the flange thickness presents additional benefits in terms of ease of transportation and storage by facilitating stacking compared to variable depth beams.

Chapter 4
Low embodied carbon design of reinforced concrete flat slabs: Development of theory

Figure 39: Nomenclature of variables used in Chapters 4 and 5

l, span of slab.
h, total depth of slab.
d, effective depth of slab.
$\frac{l}{d_{actual}}$, actual span − to − effective − depth ratio.
$\frac{l}{d_{limit}}$, limiting span − to − effective − depth ratio.
$\frac{l}{d_{limit,basic}}$, limiting span − to − effective − depth ratio unmodified for long spans and actual tensile reinforcement provided.
$\frac{l}{d_{limit,span}}$, limiting span − to − effective − depth ratio modified for long spans but not modified for the actual tensile reinforcement provided
z_{ULS}, lever arm of cracked section under plastic stress distribution.
z_{SLS}, lever arm of cracked section under elastic stress distribution.
$A_{S,req}$, area of reinforcement required for resistance.
$A_{S,crack}$, minimum area of reinforcement required for crack control.
$A_{S,min}$, minimum area of reinforcement.
$A_{S,max}$, maximum area of reinforcement.
$A_{S,prov}$, area of reinforcement provided.
ρ, required reinforcement ratio at mid − span to resist the ULS moment.
$\rho_0 = \sqrt{f_{ck}} \times 10^{-3}$, reference reinforcement ratio. See clause 7.4.2(2) of EN1992 − 1 − 1.
S_{max}, maximum spacing allowable between bars for crack control.
A_{ct}, area of concrete within the tensile zone.
M_{ULS}, design bending moment under ULS load combination.
M_{SLS}, design bending moment under SLS load combination.
$\sigma_{S,SLS}$, steel stress under SLS load combination.
f_{ck}, characteristic compressive cylinder strength of concrete at 28 days.
f_{yk}, characteristic yield strength of reinforcement, taken as 500MPa.
$f_{yd} = \frac{f_{yk}}{1.15}$, design yield strength of reinforcement. See Table 3.1 of EN1992 − 1 − 1.
$f_{ctm} = 0.30 \times f_{ck}^{2/3}$, mean value of axial tensile strength of concrete.
E_s, design value of modulus of elasticity of steel. Taken as 200MPa. See clause 3.2.7(4) of EN1992 − 1 − 1.
E_{cm}, secant modulus of elasticity of concrete. See Table 3.1 of EN1992 − 1 − 1.
$E_{c,eff}$, effective modulus of elasticity of concrete, modified to account for creep.
$\alpha = \frac{E_s}{E_{c,eff}}$, modular ratio. Taken as $\frac{3 \times E_s}{E_{cm}}$. See Appendix G for justification.
x_{1c}, depth of the neutral axis just before formation of the first crack.
ε_{cc}, compressive strain in extreme compressive fibre of the section.
ε_{ct}, tensile strain in extreme tensile fibre of the section.
ε_{st}, tensile strain in tensile reinforcement.

σ_{cc} , *compressive stress in extreme compressive fibre of the section.*

σ_{ct} , *tensile stress in extreme tensile fibre of the section.*

σ_{st} , *tensile stress in tensile reinforcement.*

F_{cc} , *resultant compressive force of compressive stresses in concrete.*

F_{ct} , *resultant tensile force of tensile stresses in concrete.*

F_{st} , *tensile force in tensile reinforcement.*

$E_{c,eff,1c}$, *effective modulus of elasticity at formation of first crack.*

$\alpha_{1c} = \dfrac{E_s}{E_{c,eff,1c}}$, *modular ratio. Taken as* 5.7. *See Appendix F for justification.*

$A_{S,TOP,COL,i}$, *top reinforcement provided in area i of column strip. See* Figure 43.

$A_{S,TOP,MID,i}$, *top reinforcement provided in area i of middle strip. See* Figure 43.

$A_{S,BOT,i}$, *bottom reinforcement provided in area i. See* Figure 44.

$V_{S,TOP,COL,i}$, *volume of additional top reinforcement provided in area i of column strip. See* Figure 43.

$V_{S,TOP,MID,i}$, *volume of additional top reinforcement provided in area i of middle strip. See* Figure 43.

$V_{S,TOP,COL,DIRj}$, *volume of additional top reinforcement provided one column strip in direction j.*

$V_{S,TOP,MID,DIRj}$, *volume of additional top reinforcement provided one middle strip in direction j.*

$V_{S,NOM,TOP,DIRj}$, *volume of nominal top reinforcement provided in whole slab in direction j.*

$V_{S,TOP,DIRj}$, *volume of total top reinforcement provided in whole slab in direction j.*

$V_{S,TOP}$, *volume of total top reinforcement provided in whole slab.*

$V_{S,BOT,i}$, *volume of additional bottom reinforcement provided in area i. See* Figure 44.

$V_{S,NOM,BOT,DIRj}$, *volume of nominal bottom reinforcement provided in whole slab in direction j.*

$V_{S,BOT,DIRj}$, *volume of total bottom reinforcement provided in whole slab in direction j.*

$V_{S,BOT}$, *volume of total bottom reinforcement provided in whole slab.*

$NSpan_{DIRj}$, *number of spans in slab in direction j.*

This is the first of two chapters dedicated to the second case study carried out as part of this project. It relates to the first area of investigation highlighted in the last section of Chapter 1: '*Development of an environmentally-conscious design practice*' and investigates the design of flat slabs to minimise their embodied carbon content.

This chapter introduces the research question, as well as the methodology and theory used to investigate it.

4.1 RESEARCH QUESTION

As explained in Chapter 1, UK building regulations currently focus only on operational aspects. Aspects relating to the '*making*' of buildings are left unconsidered. As a consequence, there is no legislative incentive for designers to reduce the embodied environmental impact of their work, and it is reasonable to assume that, as a result, contemporary buildings are designed and built with a greater embodied environmental impact than could be achieved if these issues were given more importance.

In a previous, limited, study (Thirion 2010), the author proposed to quantify the gap between the embodied environmental impact of a typical reinforced concrete building structure as designed and built, and that of an alternative design, still complying with current codes of practice and using commonly available construction methods. Due to the complexity in defining embodied environmental impact highlighted in Chapter 1, embodied carbon was used as a proxy for embodied environmental impact in the study.

The study pointed out opportunities for reducing the quantities of carbon embodied in the structure. Regarding the design of the columns, the original design was shown to use a very limited number of sections. By increasing the number of column sections from three in the original design to eight, and by assigning a section to each column based on its required capacity, the embodied carbon in the columns was reduced by as much as 60%. The study of the slabs also pointed out a number of opportunities for reduction, but had shortcomings, due in particular to a misunderstanding regarding the loading considered for the original design.

Another shortcoming of this study is that the investigation into the savings to be obtained from modifying the original design was carried out on a trial-and-error basis, and without a clear understanding of the measures which would result in a reduction in the quantities of carbon embodied in the structure. Additionally, the effect of the uncertainties in the embodied carbon intensities of materials, highlighted in Section 1.3.1 of Chapter 1, was not considered. These may however significantly limit the ability of designers to make informed decisions, or may even make it impossible altogether. Indeed, at the moment when a slab is designed, the contractor has not been appointed, and the origins of the materials to be used for construction are not known resulting in significant uncertainties regarding the carbon intensities of materials.

This study proposes to address these issues. Through a comprehensive investigation specifically into the design of reinforced concrete flat slabs, the present study aims to contribute to the understanding of how to design low environmental impact reinforced concrete building structures. This understanding is growing but is still sparse. A significant amount of work has been done on the study of the potential for reducing the initial embodied environmental impact of reinforced building structures through the use of cement replacement products. Research into the potential to reduce this impact through the *sustainable use of structural materials*, while focusing only on currently available design and construction methods, is more limited. Studies have been carried out on the relative embodied carbon content of commonly used flooring systems. Griffin (2010) compared six commonly used flooring systems. He demonstrated that significant savings – between 17% and 55% – could be obtained in the quantities of carbon embodied in the floors by appropriately selecting the type of structural system used. At the smaller scale of structural elements themselves, Wani (2011) looked into the impact of the choice of the depth of simply-supported beams on their embodied carbon content.

Such studies contribute to developing our appreciation of how to design reinforced concrete structures to minimise their embodied environmental impact. Put in perspective, they enable an understanding of the relative effects of each possible measure. Using these, designers will thus be able to balance the benefits to be obtained in terms of environmental impact from a particular design decision, with other aspects of the design: architecture, services, construction and cost. Indeed, even with incentives in place to encourage designers to reduce the environmental impact of their work, design will remain a matter of managing trade-offs and designers need appropriate tools and knowledge to make decisions.

This understanding would also be useful to carry out a review of current design practice and quantify the savings to be obtained from a more environmentally conscious practice, making the best use of commonly available design and construction techniques, as initially intended.

Flat slabs have been chosen here for investigation as they are a popular flooring system in the UK. Moreover, in the previous study by the author (Thirion 2010) mentioned above, the floors were found to concentrate a significant proportion of the total carbon embodied in the structure. As can be seen from Figure 40 and Table 6 below, which summarise the quantities of carbon embodied in the main structural elements of the building as originally designed, the floors represented more than two thirds of the total embodied carbon in the structure.

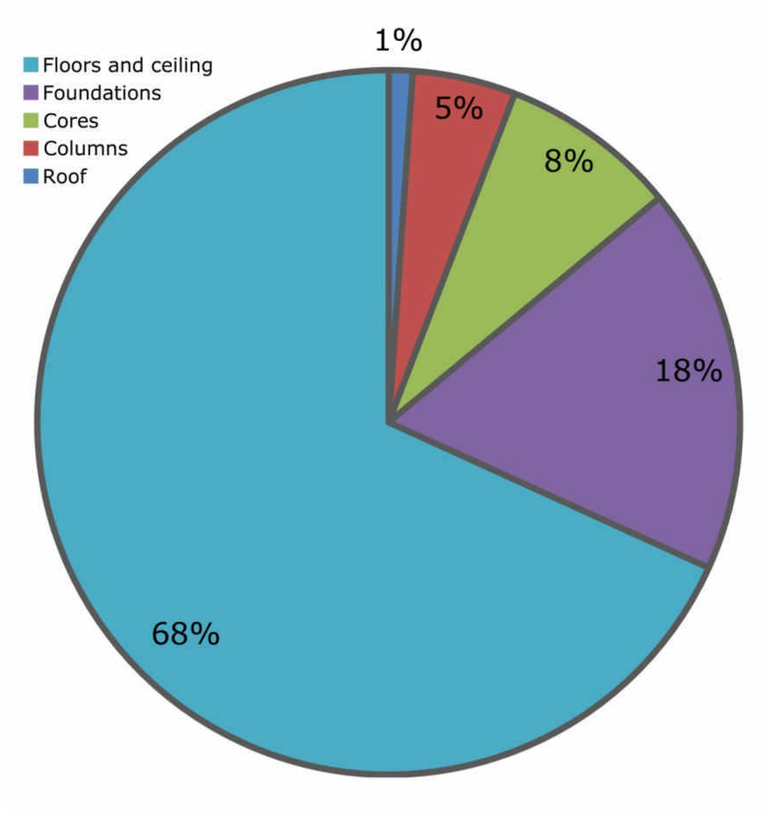


Figure 40: Distribution of embodied carbon in building structure used as case study, presented by type of structural element (Thirion 2010)

Element	**Embodied carbon**	**Embodied carbon**
	(TCO2)	**(% of total carbon embodied in structure)**
Foundations	356	18%
Columns	95	5%
Floors and ceilings	1367	68%
Cores	164	8%
Roof	22	1%
Total	**2004**	**100%**

Table 6: Distribution of embodied carbon in building structure used as case study, presented by type of structural element (Thirion 2010)

4.2 SCOPE AND METHODOLOGY

In this section, the scope of the study and the methodology adopted for investigation are described.

4.2.1 Scope

The scope of the study includes two aspects which are discussed in turn in this section: the characteristics of the slabs used for this investigation, and the aspects of flat slab design the impact on embodied carbon quantities of which is investigated.

4.2.1.1 Characteristics of the slabs used for investigation

Flat slabs are used in buildings with different end usages. The building usage considered in this study is residential. Further studies could investigate other building usages such as offices. It is however expected that the trends highlighted in this study are similar for other building usages.

The slabs studied are internal slabs supported on a square column grid. Rectangular panels of different aspect ratios could be studied, and may be found to be more efficient. This limitation in scope should not affect the trends highlighted by the present results, as they compare different cases on a similar basis. Although in residential applications the grid of columns supporting the slabs is often irregular as it is chosen to fit the internal space layout, in this study it is taken as regular in both directions, as the aim of the study is to highlight general trends. The slabs considered have at least three continuous spans in each direction. Slabs using a lesser number of spans are possible, but are less common and are hence not considered here.

Goodchild (2009) and The Concrete Centre (2007) give guidance on the spans for which flat slabs with square panels are an economic option: these spans range between 6m and 9m. This is the range of spans which is considered in this study.

4.2.1.2 Aspects investigated

In the design of flat slabs, a large number of choices need to be made by the designer. In this study, the impacts of two of these choices on the embodied carbon intensity of flat slabs are investigated: slab depth and grade of concrete.

The depth of a slab is known to have an influence on the cost of a slab, as it has a direct impact on the quantities of concrete and reinforcement required. The Concrete Centre (Goodchild 2009) has produced guidance on the selection of a slab depth at which the relative quantities of concrete and reinforcement result in the most economic slab. A similar optimal point must exist at which the embodied carbon content of a slab is minimal. The impact of the choice of the depth of a slab is hence the first aspect investigated here.

The depth of a slab is a key dimension in the design of a building due to the impact it has on the overall design of the building and the work of members of the design team other than the structural engineer. It is chosen in the early stages of the design and can often not be altered later on. It is hence important that designers are able to make an informed decision early in the process and the possibility of giving guidance to designers regarding the slab depths to target to design minimum embodied carbon flat slabs will be discussed.

Using a higher grade of concrete may result in shallower slabs due to the increased concrete modulus of elasticity, and potentially in reduced quantities of reinforcement from the increased concrete strength. However, as shall be seen below, higher concrete grades also have a larger embodied carbon intensity due to the additional quantities of cement they require. This study will investigate the effects of these two trends and conclude as to whether higher concrete grades are worth considering.

Another aspect which will be discussed in this section is the impact on the quantities of reinforcement provided, and on the resulting quantities of embodied carbon, of using a finite standardised set of bar sizes at a spacing kept constant throughout the slab, as is common practice. This aspect does not strictly fall into the investigation of current practice to which this study relates. It is more related to the same area of investigation as the study of the shape optimisation of steel I-beams presented in the two previous sections.

4.2.2 Methodology

Structural calculations in this investigation are carried out to Eurocode 2 as this is now the prevailing structural design code of practice for concrete structures in the UK.

To investigate the research question described above, a number of slabs are designed which use a range of different slab depths, concrete grades and spans. For this, one of the analysis and design methods for flat slabs described in EN1992-1-1 (BSI 2004b) is implemented in a spreadsheet format, which enables the calculation in an semi-automated manner of the minimum quantities of materials required for the design to be viable.
A detailing method is also derived which is based on requirements from the code. It also takes into account the results of a series of interviews with practicing structural engineers at Expedition Engineering, in order for the detailing to be representative of current engineering practice.

Due to the complexity associated with defining embodied environmental impact highlighted in Chapter 1, embodied carbon is used as a proxy for embodied environmental impact in the study.

These aspects are described in further detail in the following section.

4.3 THEORY

This section describes the analysis and design methods adopted for this investigation, along with the method used to detail the reinforcement to be provided in the slabs. The methods used to work out the quantities of reinforcement provided and the quantities of carbon embodied in the slabs are also described. But first the concrete cover to reinforcement used and the design loading considered are presented.

4.3.1 Concrete cover

The concrete covers used for the slabs are chosen to ensure adequate durability, bond and fire protection (Bond 2006). The cover for durability chosen is for internal elements as per BS8500 (BSI 2006b). The values of concrete cover depend on the maximum bar diameter used in the slab and are summarised in Table 7.

Maximum bar diameter	**Cover**
(mm)	**(mm)**
10	25
12	25
16	30
20	30
25	35
32	45

Table 7: Concrete covers considered depending on maximum bar diameter used in slabs

4.3.2 Design loading

The loading intensities considered are consistent with the residential building usage considered.

The slab self-weight (SW) is assessed on the basis of a 2.5tonnes/m^3 density.

The total super-imposed dead load (SDL) corresponds to the weights of the elements listed in Table 8.

An imposed load (IL) of 1.5kN/m^2 is considered.

The load combinations considered in the design of the slabs are summarised in Table 9 (BSI 2004, BSI 2006).

Element	Surface load
	(kN/m²)
Screed (75mm thick)	1.8
Ceiling (finishes and services)	0.2
Partitions	1.0
Total	**3.0**

Table 8: Breakdown of super-imposed dead loads considered

ULS combination	1.35 (SW+SDL) + 1.50 IL
SLS combination	1.00 (SW+SDL) + 0.30 IL

Table 9: Load combinations considered

4.3.3 Analysis method

Different analysis methods exist for flat slabs. These have been reviewed in detail by The Concrete Centre (The Concrete Centre 2007) and include *equivalent frame analysis*, *finite element analysis* (FEM), *yield line analysis* and *grillage analogy*. The first two methods are the most commonly used in practice.

In the equivalent frame analysis, the slab is discretised in column and middle strips, and the bending moments seen in a whole panel are distributed in each direction respectively between the two strip types. This analysis method is only usable on grids which have columns arranged along lines in at least one direction. Its main advantage is that the analysis of a slab can be done fully by hand calculations for regular grids.

The finite element method enables a more accurate assessment of the bending moments in the slab and can accommodate any irregularity in the column grid.

For the present study, the equivalent frame analysis is used for two reasons: first, the column grids considered are regular. Moreover this method is more appropriate to an automated approach, which is desirable here as a series of variations in a number of given design cases are to be successively tested.

As the grid of columns considered is regular, the bending moment coefficients for continuous beams are used to calculate the bending moments in each slab panel. These are given in Table 10 (Bond 2007), and assume that columns provide a pin support to the slabs. Interviews with practicing engineers confirmed that this assumption was common practice at Expedition Engineering.

Annex I of EN1992-1-1 (BSI 2004b) advises on the method to apportion the bending moments in whole panels between column and middle strips. Unlike BS8110 (BSI 1997), EN1992-1-1 gives ranges of permitted values of coefficients. The coefficients used in this study are those recommended in BS8110 which are acceptable to EN1992-1-1. These are summarised in Table 11.

Working out the design bending moments at eight locations in the slab – in the end and middle spans and above the first interior and interior columns, for the column and middle strips respectively, as summarised on Figure 41 – enables the design of slabs of any number of bays in each direction, provided that this is more than three continuous spans in each directions as this is a condition for the applicability of the coefficients given in Table 10.

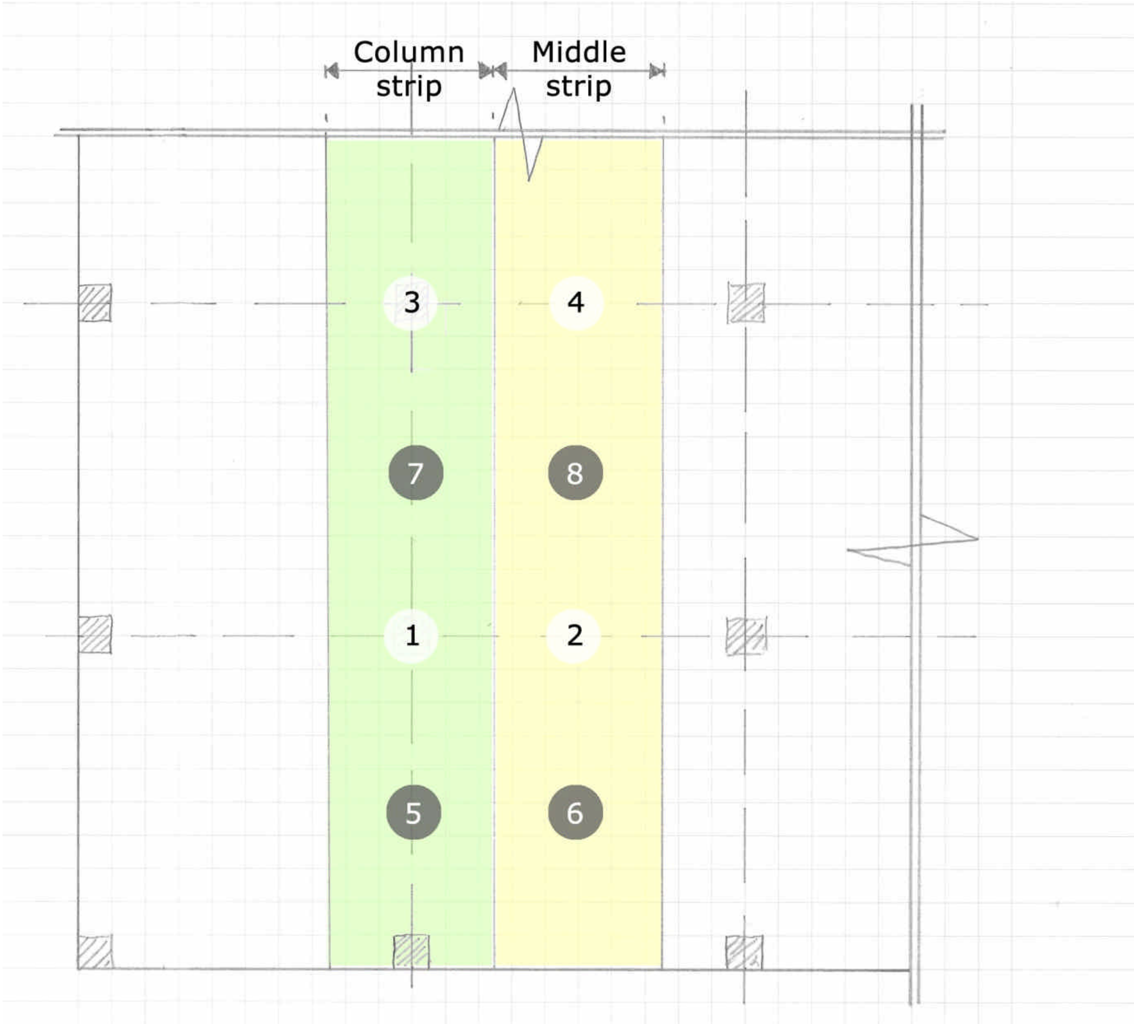
Location	End support	End span	First interior support	Interior span	Interior supports
Design moment	0	0.086Fl	- 0.086Fl	0.063Fl	- 0.063Fl

Notes:	- Valid if slab has at least three continuous spans. - A positive moment is a sagging moment. Negative is a hogging moment. - F is the total load on one panel under the ULS load combination. - l is the effective span. - These coefficients are based on a 20% redistribution at the supports and no decrease in span moment.

Table 10: Bending moment coefficients for flat slabs (Bond 2007)

Type of bending moment	**Column strip**	**Middle strip**
Hogging	75%	25%
Sagging	55%	45%

Table 11: Column and middle strip coefficients used



1	Column strip hogging moment at first interior support
2	Middle strip hogging moment at first interior support
3	Column strip hogging moment at interior support
4	Middle strip hogging moment at interior support
5	Column strip sagging moment in end span
6	Middle strip sagging moment in end span
7	Column strip sagging moment in interior span
8	Middle strip sagging moment in interior span

Figure 41: Plan of slab showing the eight points at which design bending moments need to be calculated

4.3.4 Design method

In this section, the ULS and SLS design checks carried out on each slab are described.

4.3.4.1 ULS flexural design

The ULS design of each section of a slab uses the simplified rectangular stress block from EN1992-1-1 (BSI 2004b).

4.3.4.2 SLS design

The SLS design of flat slabs comprises two aspects: limiting deflections and limiting cracking. These are reviewed in turn here.

Recommended deflection limits are specified by EN1992-1-1 (BSI 2004b) as span/250 under the quasi-permanent load combination and span/500 for post-construction deflections under quasi-permanent loads. Deflections may be calculated and compared to the above limiting values. Alternatively, deemed-to-satisfy simplified rules on the slab span-to-effective-depth ratio can be used as detailed in Clause 7.4.2 of the code.

Unless project specific design criteria impose tighter limitations on deflections than those mentioned above, span-to-effective-depth rules are normally used by designers as they are simpler to implement than actually calculating the deflections. A full calculation of the deflections occurring in a slab requires a large number of assumptions to be made, and is a more time- and calculation-intensive process. The slabs considered here are assumed not to be subject to tighter deflection limits than the basic ones specified by the code, and hence span-to-depth ratios are used to check deflections.

Similarly, cracking can be checked by actual calculations or by using deemed-to-satisfy rules which impose a limit on either the maximum bar diameter used, or on the maximum spacing between bars. The latter approach, described in Clause 7.3.3 of EN1992-1-1 (BSI 2004b), is used in the present study to ensure crack widths are kept below 0.3mm as per Table NA.4 of the National Annex to EN1992-1-1 (BSI 2004c).

4.3.4.3 Summary of ULS and SLS design checks considered

The checks carried out on each slab are described in detail in Figure 42. For a definition of the variables used in Figure 42, please refer to the nomenclature given in Figure 39 at the beginning of this chapter.

ULS design: resistance

The check below applies to the calculation of both hogging and sagging reinforcement required for resistance.

$$k = \frac{M_{ULS}}{1000\ d^2\ f_{ck}}$$

$$z_{ULS} = 0.5\ d\left(1 + \sqrt{1 - 3.53 \times k}\right) \leq 0.95\ d$$

$$A_{S,req} = \frac{M_{ULS}}{f_{yd}\ z_{ULS}}$$

$$\boldsymbol{CHECK}: A_{S,prov} \geq A_{S,req}$$

SLS design: deflections

The check presented below applies to the sections located at midspan between columns. It is derived as per clause 7.4.2(2) of EN 1992-1-1 (BSI 2004b).

Basic span-to-effective depth ratio

$If\ \rho \leq \rho_0$:

$$\frac{l}{d_{limit,basic}} = 1.2\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right]\ (Expr. 7.16a\ of\ EN1992-1-1)$$

$Else$:

$$\frac{l}{d_{limit,basic}} = 1.2\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho}\right]\ (Expr. 7.16b\ of\ EN1992-1-1)$$

Span-to-effective depth ratio modified for long spans

$If\ l \leq 8.5m$:

$$\frac{l}{d_{limit,span}} = \frac{8.5}{l} \times \frac{l}{d_{limit,basic}}$$

$Else$:

$$\frac{l}{d_{limit,span}} = \frac{l}{d_{limit,basic}}$$

Span-to-effective depth ratio modified for provided tensile reinforcement

$$M_{SLS} = M_{ULS} \times \frac{p_{SLS}}{p_{ULS}}$$

$$z_{SLS} = d + \frac{\left[2 \times \alpha \times\ A_{S,prov} - \sqrt{4 \times \alpha^2 \times\ A_{S,prov}{}^2 + 8000 \times \alpha \times\ A_{S,prov}\ d}\right]}{6000}\quad with\ \alpha = \frac{600}{E_{cm}}$$

$(see$ Appendix G $for\ details\ of\ calculations)$

$$\sigma_{S,SLS} = \frac{M_{SLS}}{z_{SLS}\ A_{S,prov}}$$

$$\frac{l}{d_{limit}} = \frac{310}{\sigma_{S,SLS}} \times \frac{l}{d_{limit,span}}$$

$$\leq 1.5\frac{l}{d_{limit,span}}\quad (Note\ 5\ of\ Table\ NA5\ of\ NA\ to\ EN1992-1-1\ (BSI\ 2004c))$$

$$\boldsymbol{CHECK}: \frac{l}{d_{actual}} \leq \frac{l}{d_{limit}}$$

Figure 42: Summary of checks carried out in the design of a flat slab, as per EN1992-1-1

SLS design: crack control

The checks presented below apply to any section of the slab.

Minimum reinforcement area – Clause 7.3.2(2) of EN1992-1-1 (BSI 2004b)

$$A_{ct} = 1000\left(h - \frac{500\,h^2 + A_S\,\alpha_{1c}\,d}{1000\,h + A_S\,\alpha_{1c}}\right). See\ Appendix\ F\ for\ details\ of\ calculations.$$

$$A_{S,crack} = \frac{0.4\,f_{ctm}A_{ct}}{f_{yk}}$$

$\boldsymbol{CHECK}: A_{S,prov} \geq A_{S,crack}$

Maximum spacing between bars

$$S_{max} = 500 - \frac{\sigma_{S,SLS}}{0.8}\ (Table\ 7.3N\ of\ EN1992-1-1\ (BSI\ 2004b)$$

$\boldsymbol{CHECK}: S_{prov} \leq S_{max}$

Note: The recommendations on maximum spacing of bars in solid slabs given in Clause 9.3.1.1(3) of EN1992-1-1 (BSI 2004b) are necessarily complied with as the reinforcement in the slabs uses a basic mesh spacing of 200mm – see Section 4.3.5.5 of this report.

Minimum and maximum reinforcement areas

The checks presented below apply to any section of the slab.

Minimum reinforcement area

$$A_{S,min,1} = 1.2\,A_{S,req}\ (Note\ of\ Clause\ 9.3.1.1(1) of\ EN1992-1-1\ (BSI\ 2004b))$$

$$A_{S,min,2} = \frac{0.26\,f_{ctm}\,1000\,d}{f_{yk}} \geq 0.0013\ \times 1000\ \times b\ (Clause\ 9.2.1.1(1), Expression\ (9.1)\ of\ EN1992-1-1\ (BSI2004b))$$

$$A_{S,min} = MIN\left(A_{S,min,1}\ ;\ A_{S,min,2}\right)$$

$\boldsymbol{CHECK}: A_{S,prov} \geq A_{S,min}$

Maximum reinforcement area

$$A_{S,max} = 0.04 \times h\ \times 1000\ (Note\ of\ Clause\ 9.2.1.1(3) of\ EN1992-1-1\ (BSI\ 2004b))$$

$\boldsymbol{CHECK}: A_{S,prov} \leq A_{S,max}$

Figure 42 (Continued): Summary of checks carried out in the design of a flat slab, as per EN1992-1-1

4.3.5 Reinforcement detailing assumptions

This section describes the detailing assumptions considered in the design of the slabs. These are based on requirements from the code, as well as interviews with structural engineers at Expedition Engineering. The interviews aimed to understand the way in which practicing engineers detail flat slab reinforcement in practice in order to take it into account in the present study.

4.3.5.1 Curtailment of longitudinal tension reinforcement

As is common with Eurocode, the rule for curtailing longitudinal tension reinforcement is given as a first principle rule. Clause 9.2.3 of EN1992-1-1 (BSI 2004b) states that 'sufficient reinforcement should be provided at all sections to resist the envelope of the acting tensile force'.

This rule is not very practical to implement in a case where the design bending moments are assessed using the coefficients for continuous beams as is the case here. Clause 3.12.10.3 of BS8110 (BSI 1997) gives simplified detailing rules for cases where the slab is designed predominantly for uniformly distributed loads, the design has been carried out for the single load case of maximum design loads on all spans, and the spans are approximately equal. The slabs considered in this study comply with these requirements, and the rules given in Figure 3.25 of BS8110 (BSI 1997) are used to curtail the reinforcement provided in the slabs.

4.3.5.2 Hogging reinforcement over internal columns

Clause 9.4.1(2) of EN1992-1-1 (BSI 2004b) requires that half of the top reinforcement required in the whole panel be placed in a width of a quarter of the width of a panel centred on the column. In the case under consideration where the slab panels are square, and hence the column strip width is equal to half the width of the panel, this requirement is equivalent to increasing the area of hogging reinforcement required by a 4/3 factor in the central half width of the column strip.

From interviews with practicing engineers, it was found that it is common practice to provide the same reinforcement in the whole width of the column strip, rather than having two more lightly reinforced bands around a heavily reinforced central half band. In order for the analysis to be representative of current design practice, this detailing method is used in the design of the slabs.

4.3.5.3 Supplemented mesh or replacement reinforcement

Two main methods are available to detail the reinforcement in a slab. The first one consists of providing a nominal mesh, of an area close to the minimum area of reinforcement required by the code, in the whole slab in the top and bottom layers, and to provide additional loose bars where required. The other method consists of replacing the basic mesh with a heavier one where the basic mesh is insufficient.

Both options are used in practice and interviews with practicing engineers did not highlight one method as necessarily superior to the other, the ultimate decision being down to the contractor's preferred option. The supplemented mesh method is used here to detail the slabs.

4.3.5.4 Basic bar spacing

Detailing of the slab reinforcement has been carried out with a 200mm basic spacing between bars.

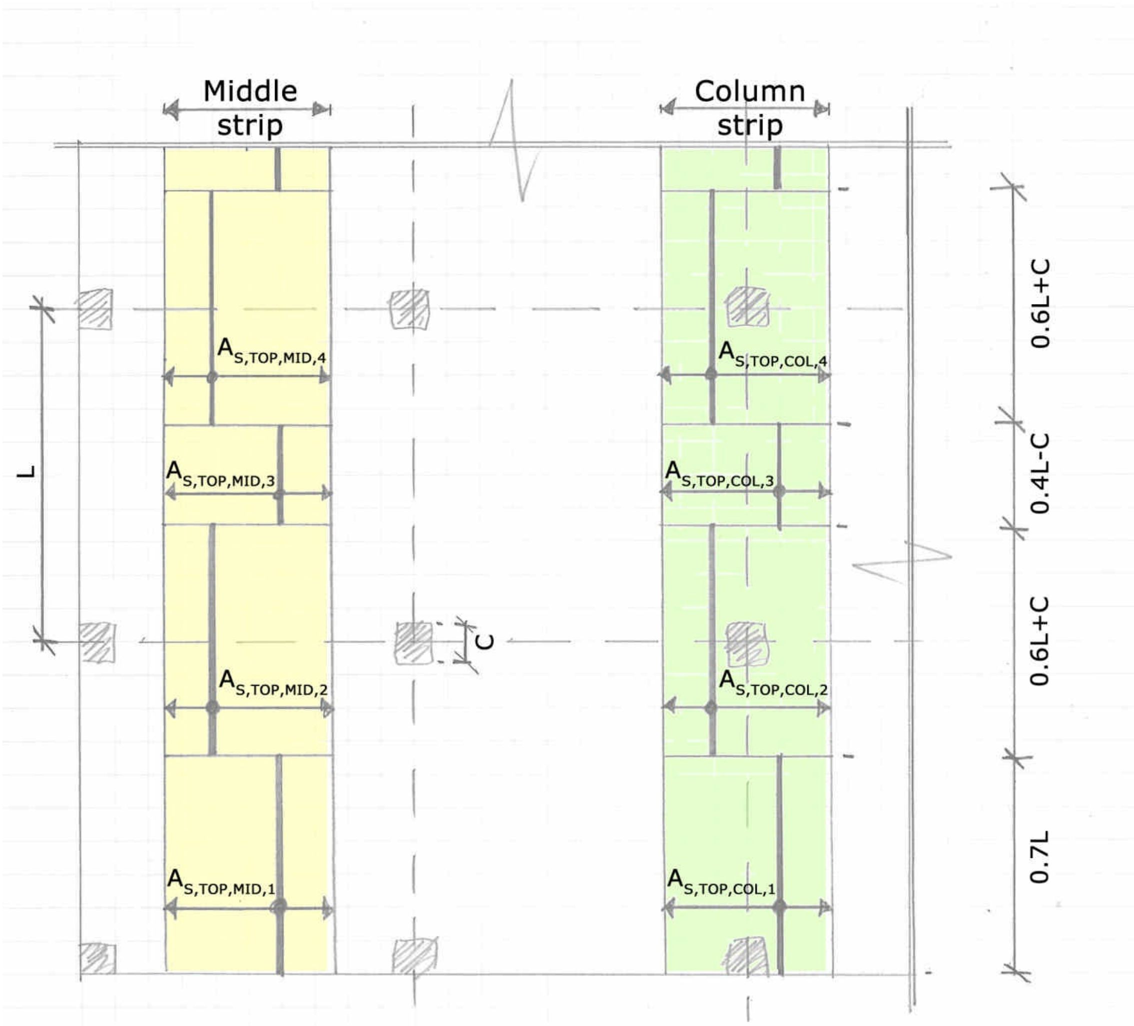
4.3.5.5 Minimum reinforcement

As highlighted in Figure 42 which summarises the design method, a minimum amount of reinforcement is required to be provided in the whole slab. From discussions with practicing engineers, it emerged that most engineers would not specify a lighter mesh than 10mm diameter bars spaced at 200 (denoted H10@200 in the rest of this study), some being even reluctant to use anything smaller than H12@200.

This choice comes partly from the fact that smaller diameter bars are so flexible that they become hard to handle on site, and partly from an impression that a lighter mesh would not be sufficient to adequately prevent cracking due to concrete shrinkage. This was taken into account in the design of the slabs, where a minimum mesh of H10@200 was provided everywhere as a minimum.

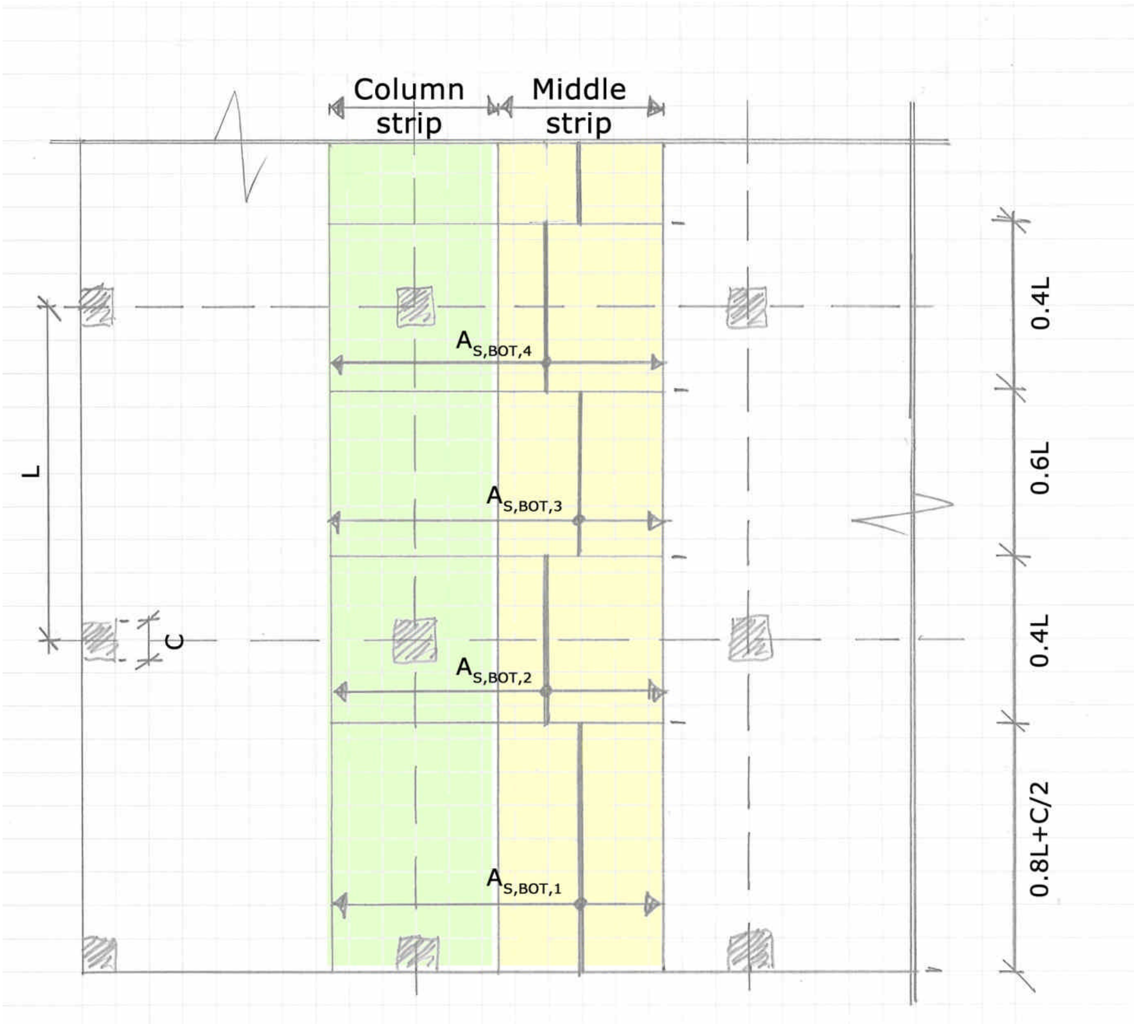
4.3.5.6 Summary of reinforcement detailing method

The reinforcement detailing method adopted is summarised on Figure 43 and Figure 44.



$A_{S,TOP,COL1}$	Minimum reinforcement to be provided
$A_{S,TOP,COL2}$	Reinforcement to resist hogging moment in column strip at first interior support - increased as per Section 4.2.2.2.8.2
$A_{S,TOP,COL3}$	Minimum reinforcement to be provided
$A_{S,TOP,COL4}$	Reinforcement to resist hogging moment in column strip at interior support - increased as per Section 4.2.2.2.8.2
$A_{S,TOP,MID1}$	Minimum reinforcement to be provided
$A_{S,TOP,MID2}$	Reinforcement to resist hogging moment in middle strip at first interior support - increased as per Section 4.2.2.2.8.2
$A_{S,TOP,MID3}$	Minimum reinforcement to be provided
$A_{S,TOP,MID4}$	Reinforcement to resist hogging moment in middle strip at interior support - increased as per Section 4.2.2.2.8.2

Figure 43: Plan view of slab summarising the detailing method used for top reinforcement



$A_{S,BOT1}$	Reinforcement to resist sagging moment in end span in column strip
$A_{S,BOT2}$	Provide 40% of the maximum between $A_{S,BOT1}$ and $A_{S,BOT3}$
$A_{S,BOT3}$	Minimum reinforcement to be provided
$A_{S,BOT4}$	Provide 40% $A_{S,BOT3}$

Figure 44: Plan view of slab summarising the detailing method used for bottom reinforcement

4.3.6 Material quantities

As explained above in Section 4.3.3, knowing the design bending moments at eight points in the slab means that the bending moments are known in the whole slab whatever its size. This consequently also applies to the provided reinforcement as can be seen from Figure 43 and Figure 44.

The total volumes of top reinforcement provided in a slab can be estimated by using the following equations. For a definition of the variables used, the reader is referred to Figure 39 at the beginning of this chapter.

$$V_{S,TOP,COL,DIR1} = 2 \times V_{S,TOP,COL,1} + 2 \times V_{S,TOP,COL,2} + (NSpan_{DIR1} - 2) \times V_{S,TOP,COL,3} + (NSpan_{DIR1} - 3) \times V_{S,TOP,COL,4}$$

$$V_{S,TOP,MID,DIR1} = 2 \times V_{S,TOP,MID,1} + 2 \times V_{S,TOP,MID,2} + (NSpan_{DIR1} - 2) \times V_{S,TOP,MID,3} + (NSpan_{DIR1} - 3) \times V_{S,TOP,MID,4}$$

$$V_{S,TOP,DIR1} = V_{S,NOM,TOP,DIR1} + NSpan_{DIR2} \times \left(V_{S,TOP,COL,DIR1} + V_{S,TOP,MID,DIR1}\right)$$

$$V_{S,TOP,COL,DIR2} = 2 \times V_{S,TOP,COL,1} + 2 \times V_{S,TOP,COL,2} + (NSpan_{DIR2} - 2) \times V_{S,TOP,COL,3} + (NSpan_{DIR2} - 3) \times V_{S,TOP,COL,4}$$

$$V_{S,TOP,MID,DIR2} = 2 \times V_{S,TOP,MID,1} + 2 \times V_{S,TOP,MID,2} + (NSpan_{DIR2} - 2) \times V_{S,TOP,MID,3} + (NSpan_{DIR2} - 3) \times V_{S,TOP,MID,4}$$

$$V_{S,TOP,DIR2} = V_{S,NOM,TOP,DIR2} + NSpan_{DIR1} \times \left(V_{S,TOP,COL,DIR2} + V_{S,TOP,MID,DIR2}\right)$$

$$V_{S,TOP} = V_{S,TOP,DIR1} + V_{S,TOP,DIR2}$$

The total volumes of bottom reinforcement provided in a slab can be estimated by using the following equations. For a definition of the variables used, the reader is referred to Figure 39 at the beginning of this chapter.

$$V_{S,BOT,DIR1} = V_{S,NOM,BOT,DIR1} + NSpan_{DIR2} \times \left(2 \times V_{S,BOT,1} + 2 \times V_{S,BOT,2} + (NSpan_{DIR1} - 2) \times V_{S,BOT,3} + (NSpan_{DIR1} - 3) \times V_{S,BOT,4}\right)$$

$$V_{S,BOT,DIR2} = V_{S,NOM,BOT,DIR2} + NSpan_{DIR1} \times \left(2 \times V_{S,BOT,1} + 2 \times V_{S,BOT,2} + (NSpan_{DIR2} - 2) \times V_{S,BOT,3} + (NSpan_{DIR2} - 3) \times V_{S,BOT,4}\right)$$

$$V_{S,BOT} = V_{S,BT,DIR1} + V_{S,BOT,DIR2}$$

4.3.7 Embodied carbon intensities

Due to the complexity in defining embodied environmental impact highlighted in Chapter 1, in this study, embodied carbon is used as a proxy for embodied environmental impact. Data on embodied carbon intensities of building materials is taken from the Inventory of Carbon and Energy (ICE) (Hammond 2011) developed at Bath University. It is currently considered to be the main reference in the domain of embodied carbon and energy in construction materials. The ICE covers most of the common construction materials and has been developed from an extensive literature review on the topic. It is mainly relevant to the UK and uses a cradle-to-site approach (Figure 6).

The ICE database gives best estimate values. As explained in Chapter 1, Section 1.3.1, there are significant uncertainties associated with such figures. In the case of concrete and steel, the authors of the database estimate these uncertainties to be as much a ±30%. In the following study, these uncertainties will be taken into account and their effect on the results obtained using best estimate values will be assessed.

The intensities of the main concrete types used in this study and of steel rebar are listed in Table 12.

Concrete type	**Embodied carbon** (kgCO2/kg)
C28/35	0.120
C30/37	0.126
C32/40	0.132
C35/45	0.139
C40/50	0.151
Steel - rebar	1.400

Table 12: Carbon intensities of materials used in study

This concludes the first of the two chapters of this thesis dedicated to the low embodied carbon design of reinforced concrete flat slabs. The chapter presented the research question considered, as well as the methodology and theory used to investigate it. The next chapter is dedicated to the presentation, interpretation and discussion of the results obtained.

Chapter 5
Low embodied carbon design of reinforced concrete flat slabs: Results and interpretation

5.1 INTRODUCTION

In this chapter, the methodology and theory presented in Chapter 4 are used to carry out two investigations which respectively look into the influence of respectively the slab depth and the choice of concrete grade on the embodied carbon content of flat slabs. They are followed by a discussion about the influence of using a standardised finite set of bar sizes and spacings on the quantities of reinforcement used in slabs and on their embodied carbon content.

5.2 INFLUENCE OF SLAB DEPTH ON EMBODIED CARBON CONTENT OF SLABS

As previously explained, there exists an optimal slab depth at which the relative quantities of concrete and reinforcement result in the most economic slab (Goodchild 2009). A similar optimal point must exist at which the embodied carbon of flat slabs is minimal. This is the aspect investigated in this section.

The investigation has two main aims: the first one is to gain an understanding of a general trend regarding the influence of the choice of the depth of a slab on its embodied carbon content. The second aim is to understand whether it is possible to define a slab depth, or at least a range of depths, as optimal, given the large uncertainties in the embodied carbon intensities of the two materials used in reinforced concrete that exist when the slab is designed, as highlighted previously.

This section is organised in five sub-sections: the characteristics of the slabs used for investigation are first presented. The reinforcement provided in each slab is then described, followed by a presentation of the results obtained for a given set of slabs. Two sensitivity studies follow: the first one looks into the impact of the uncertainties in embodied carbon intensities of concrete and steel rebar on the optimal slab depths. The second study investigates the effect of the size of a slab in plan on the depth resulting in the least carbon intensive design.

5.2.1 Slabs characteristics

This section presents the characteristics of the slabs considered for investigation. It covers the spans and the range of slab depths considered for each span, the grade of concrete used and the size of the slab considered for each span.

5.2.1.1 Spans considered

As previously discussed, the spans for which flat slabs are an economic option range between 6.0m and 9.0m (Goodchild 2009, The Concrete Centre 2007). Three spans are considered in this study, which cover this range at respectively 6.0m, 7.5m and 9.0m.

5.2.1.2 Slab depths considered

The slab depths considered range from a depth corresponding to the basic span-to-effective-depth ratio given in BS8110 (BSI 1997), unmodified for the actual tensile reinforcement provided, and the depth at which the slab fails in deflection.

The proposed upper-bound is obviously not a depth a designer would choose, but it gives the maximum depth above which there is absolutely no reason to go. The basic span-to-depth ratio for flat slabs given in BS8110 (BSI 1997) is 0.9x26=23.4 according to Clause 3.7.8 and Table 3.9 of the code. The maximum depths considered for each of the three spans used in this study are summarised in Table 13.

Span	Effective depth	Cover	Maximum bar diameter	Max. slab thickness considered
(m)	**(mm)**	**(mm)**	**(mm)**	**(mm)**
6	6000/23.4=257	25	16	300
7.5	7500/23.4=321	30	20	375
9	9000/23.4=385	35	25	450

Table 13: Maximum slab depths considered depending on span

5.2.1.3 Concrete grade used

The choice of concrete grade used for investigation should have little impact as the slabs are compared on the basis that they all use the same concrete grade. The results presented here are for flat slabs using a C30/37 concrete grade. This choice is made on the basis that it is currently the most commonly used concrete grade for flat slabs.

5.2.1.4 Slab size

For this first study, the sizes of the slabs used for investigation are chosen to approach those of a 50mx25m building, dimensions which are thought to be representative of a medium size residential building. The number of spans chosen for each of the three span dimensions considered are summarised in Table 14.

Span	Number of spans in direction 1	Number of spans in direction 2	Actual slab dimensions
(m)	(-)	(-)	(m2)
6	9	4	54x24
7.5	7	3	52.5x22.5
9	6	3	54x27

Table 14: Slabs dimensions used for investigation

5.2.2 Reinforcement provided

The analysis, design and detailing methods described in Section 4.3 are used to work out the reinforcement to be provided in the slabs. The reinforcement provided in each slab is presented in Table H 1 to Table H 3 in Appendix H.

5.2.3 Results

In this section, for each of the three spans considered, the quantities of steel reinforcement provided in each slab are presented, followed by a presentation of the corresponding quantities of embodied carbon.

For each slab depth, three quantities of reinforcement are given:

- *Detailing case a*: The quantity of reinforcement to be provided when using the detailing rules presented in Section 4.3.5, and by detailing using the usual set of bar sizes used in reinforcement detailing: H10, H12, H16, H20 and H25. This reinforcement weight corresponds to the reinforcement given in Table H 1 to Table H 3 in Appendix H.
- *Detailing case b*: The quantity of reinforcement theoretically required. This quantity corresponds to the quantities of rebar to be provided when the detailing rules presented in Section 4.3.5 are used, but assuming that bar sizes can be taken from a continuous range of sizes, and that the spacings between bars can be freely adjusted. Compared to the first quantity given, it excludes any extra reinforcement weight which would be due to the choice of a spacing of 200mm between bars, or to the fact that only a finite number of bar sizes is used.
- *Detailing case c*: The quantity of reinforcement theoretically required but not including any minimum reinforcement. This quantity corresponds to the reinforcement of detailing case b, but does not include the minimum areas of reinforcement required by the design code as detailed in Figure 42.

The two last quantities given are purely theoretical cases: detailing case b would require an infinite number of bar sizes to be available. Slabs using the reinforcement of detailing case c would not comply with code requirements as they may undergo excessive cracking due to shrinkage and thermal effects. These two quantities are included here as they are useful in the interpretation of the quantities of reinforcement obtained using the detailing rules adopted for the study. Detailing case b is also useful to understand whether a given result may be due to the particular detailing method adopted in this investigation.

5.2.3.1 6.0mx6.0m slab

This section presents the results obtained for flat slabs supported on a 6.0m by 6.0m column grid.

5.2.3.1.1 Steel reinforcement quantities

Figure 45 presents the three quantities of reinforcement described above as a function of the slab depth for slabs supported on a 6.0mx6.0m column grid.

The curve obtained for detailing case c is the closest to the pure mechanical understanding of reinforced concrete design: as the slab depth decreases, the total amount of reinforcement required increases; and as the slab depth approaches its minimum viable depth, the rate of increase becomes higher.

Detailing case b, which takes into account the requirement to provide minimum reinforcement shows a different picture: as can be seen from the equations in Figure 42, the minimum reinforcement required increases as the depth of the slab increases. The curve obtained for detailing case b shows that this requirement for reduced minimum reinforcement as the slab depth decreases compensates for the additional reinforcement required until a depth of 225mm is reached. When this depth is attained, the quantities of required reinforcement rise more sharply, and the reduction in minimum reinforcement associated with the reduction in slab depth is not sufficient to compensate.

The fact that the quantities of reinforcement remain almost constant until a 225mm slab depth is reached means that even without calculating the embodied carbon corresponding to each slab depth, it can be concluded that the least carbon intensive slab uses a depth comprised between 225mm and 205mm.

The curve obtained for detailing case a follows a profile largely similar to that obtained for case b, but shifted upwards, due to the fact that the reinforcement needs to be rationalised, which results in an increase in the quantities of reinforcement. The main additional difference between the two profiles is the reduction in reinforcement between a 300mm and a 275mm deep slab in detailing case a: this finds its explanation in a step change in the nominal reinforcement provided from H12@200 to H10@200.

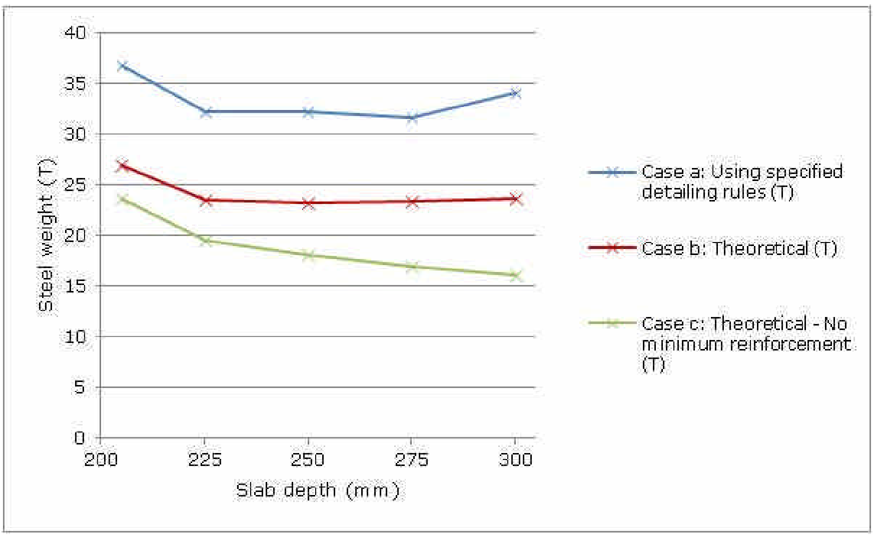


Figure 45: Weight of steel reinforcement as a function of the slab depth – 6.0mx6.0m column grid– 9x4 spans – C30/37

5.2.3.1.2 Embodied carbon quantities

Figure 46 shows the quantities of embodied carbon in the slabs as a function of the slab depth for detailing cases a and b. These quantities are based on the best estimates of the carbon intensities for concrete and rebar given in Section 4.3.7. In both detailing cases, the shallowest slab depth viable, 205mm, is found to result in the least carbon intensive design. The fact that the same trend is observed for both detailing cases shows that the result obtained is not simply due to the chosen detailing method.

Figure 47 presents the percentage increase in embodied carbon compared to the depth resulting in the least carbon intensive design. This shows an important difference between slabs using larger depths and the optimal depth. It can be noted that the relative increase between two adjacent depths tested reduces as the optimal depth is approached, and the difference in embodied carbon content between a 225mm thick slab and a 205mm thick slab is found to be only of a few percent. This finds its explanation in the sharper increase in steel quantities to be provided as the slab become shallower, as made obvious on Figure 45. In this range of shallow slab depths, the benefits of using less concrete are almost counterbalanced by the increase in steel quantities to be provided.

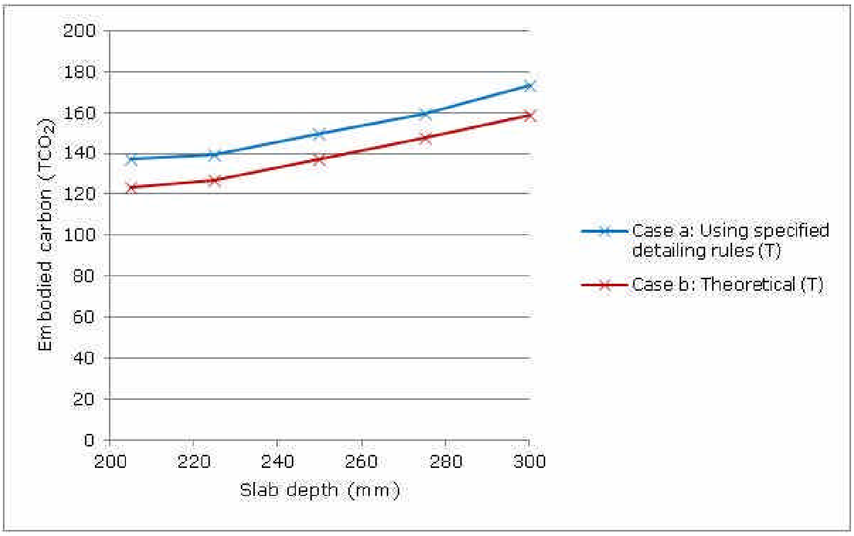


Figure 46: Embodied carbon as a function of the slab depth – 6.0mx6.0m column grid – 9x4 spans – C30/37

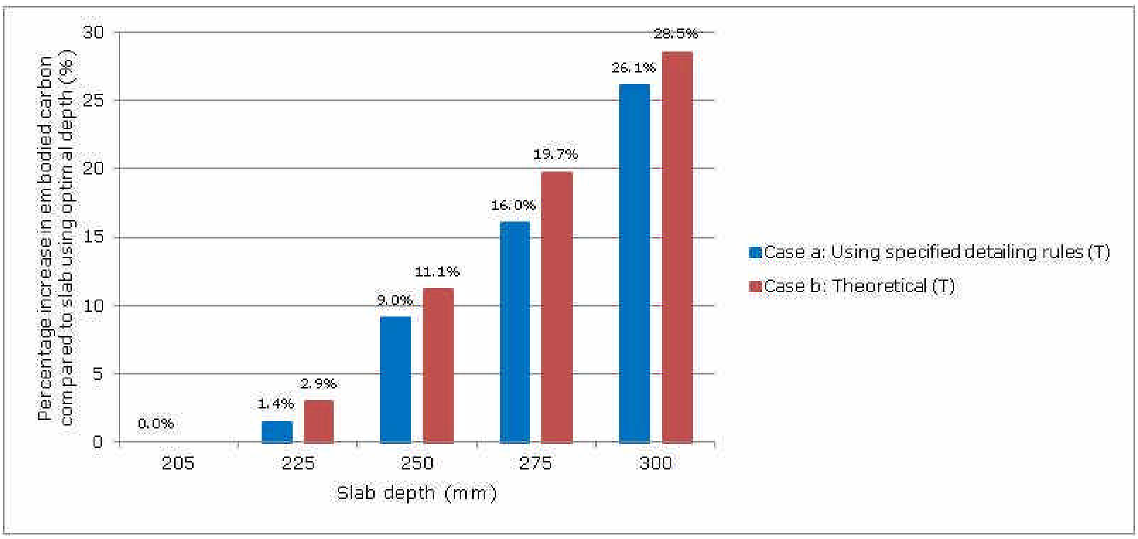


Figure 47: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth - 6.0mx6.0m column grid – 9x4 spans – C30/37

5.2.3.2 7.5mx7.5m slab

This section presents the results obtained for flat slabs supported on a 7.5m by 7.5m column grid.

5.2.3.2.1 Steel reinforcement quantities

Figure 48 presents the three quantities of reinforcement described previously as a function of the slab depth for slabs supported on a 7.5mx7.5m column grid.

The quantities of reinforcement for detailing cases b and c are found to follow a similar trend as presented above for the slabs supported on a 6.0mx6.0m column grid.

Here, the curve for detailing case c does not follow that for case b as closely as it does for the slabs supported on a 6.0mx6.0m column grid. First, the quantities of steel reinforcement provided are found to reduce between the case where a slab depth of 300mm is used and that where a slab depth of 275mm is used. This reduction has two origins: the nominal reinforcement required reduces from H12@200 to H10@200 between these two depths, resulting in a significant saving in reinforcement. Additionally, as can be seen in Table H 2, the reinforcement provided on top of the nominal reinforcement does not increase in most of the slab between the two depths. This is due to the fact that a larger amount of reinforcement than strictly required is provided in the 300mm thick slab as only a finite number of bar sizes and a constant spacing between bars are used. The additional reinforcement provided hence works at a low rate in the 300mm slab, and is hence still found to be adequate in the 275mm slab despite the reduction in slab depth and in nominal reinforcement.

The second difference between the two curves is that the quantity of reinforcement in detailing case a then barely changes between the depth of 275mm and the minimum viable depth of 260mm. This is once again due to the particularly low rate of work of the additional reinforcement provided in the 300mm thick slab, which is found to be adequate for a 260mm slab as shown in Table H 2.

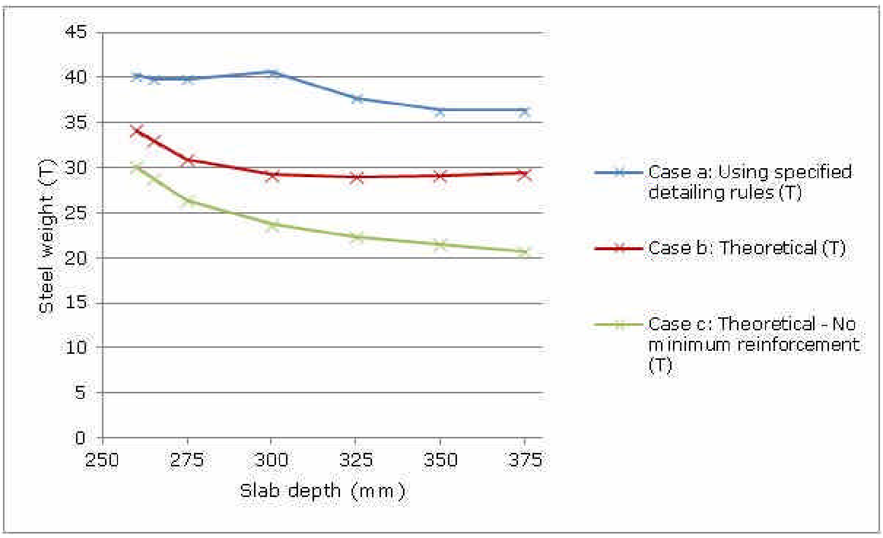


Figure 48: Weight of steel reinforcement as a function of the slab depth – 7.5mx7.5m column grid – 7x3 spans – C30/37

5.2.3.2.2 Embodied carbon quantities

Figure 49 shows the quantities of embodied carbon in the slabs as a function of the slab depth for detailing cases a and b, based on the best estimates of the carbon intensities for concrete and rebar given in Section 4.3.7. In this case again, the shallowest slab is found to be the least carbon intensive for both detailing cases a and b. Once again, the fact that the same trend is observed for both detailing cases shows that the result obtained is not simply due to the chosen detailing method.

Figure 50 presents the percentage increase in embodied carbon compared to the depth resulting in the least carbon intensive design. Again, an important difference between slabs using large depths and the optimal depth is shown and the savings obtained reduce as the optimal depth is approached: here the difference in embodied carbon content between a 275mm thick slab and a 260mm thick slab is only of a few percents.

The results obtained using detailing case a show a significant difference between the 300mm thick slab and the 275mm thick slab. As explained above, this difference comes from a reduction in the nominal reinforcement to be provided between the two depths and from the fact that the provided reinforcement only uses a finite number of bar sizes. This result points out the potential impact on the carbon intensity of a slab of detailing using only a finite set of bar sizes.

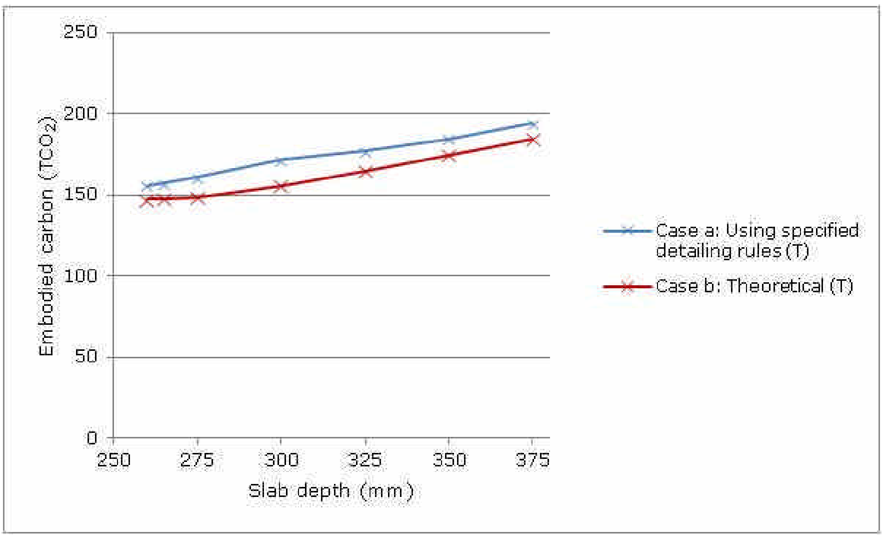


Figure 49: Embodied carbon as a function of the slab depth – 7.5mx7.5m column grid – 7x3 spans – C30/37

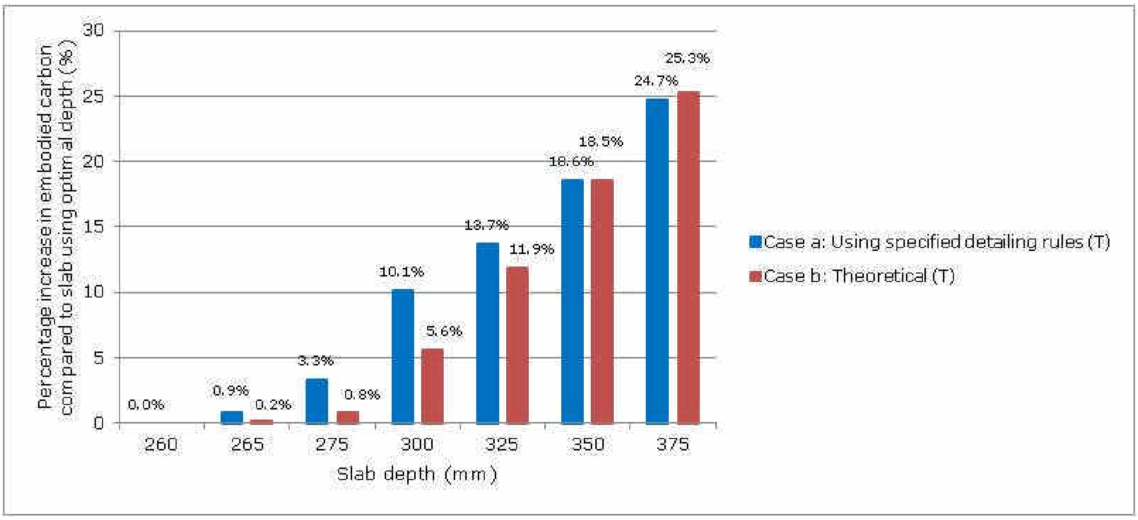


Figure 50: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth - 7.5mx7.5m column grid – 7x3 spans – C30/37

5.2.3.3 9.0mx9.0m slab

This section presents the results obtained for flat slabs supported on a 9.0m by 9.0m column grid.

5.2.3.3.1 Steel reinforcement quantities

Figure 51 presents the three quantities of reinforcement described previously as a function of the slab depth for slabs supported on a 9.0mx9.0m column grid.

The quantities of reinforcement for detailing cases b and c are found to follow a similar trend as presented above for the slabs supported on a 6.0mx6.0m column grid.

Here again, the curve for detailing case c does not follow that for case b as closely as it does for the slabs supported on a 6.0mx6.0m column grid. The main difference comes from the fact that in detailing case c, a significant reduction in steel quantities occurs between depths of 425mm and 400mm. This is due to a step change in the nominal reinforcement provided which reduces from H16@200 to H12@200 resulting in a significant saving in steel weight. Another difference to point out is the fact that, as the minimum viable slab depth is approached, reinforcement quantities start increasing more markedly in detailing case a than in detailing case b. This is again due to the fact that a finite set of bar sizes is used.

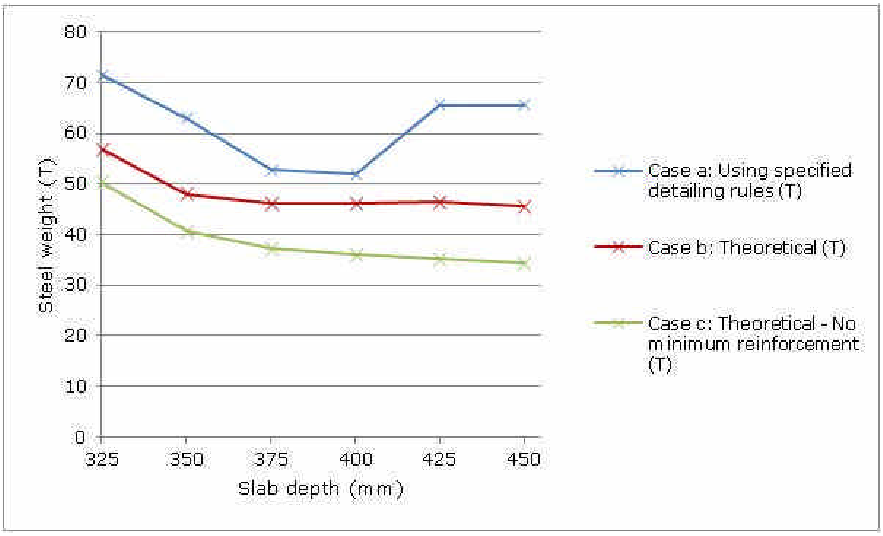


Figure 51: Weight of steel reinforcement as a function of the slab depth – 9.0mx9.0m column grid – 6x3 spans – C30/37

5.2.3.3.2 Embodied carbon quantities

Figure 52 shows the quantities of embodied carbon in the slabs as a function of the slab depth for detailing cases a and b. Figure 53 presents the percentage increase in embodied carbon compared to the depth resulting in the least embodied carbon design for detailing cases a and b respectively.

For both detailing assumptions, the shallowest slab depth is not found to result in the least carbon intensive design. Instead, a slightly larger depth is found to yield better results. This is explained by the fact that the increase in reinforcement to be provided, as the minimum viable slab depth is approached, results in an increase in embodied carbon larger than the reduction obtained from the reduction in concrete volume.

The optimal depth in detailing case a, 375mm, is larger than that obtained for detailing case b, 350mm. This is due to the more significant increase in steel observed in Figure 51 in detailing case a when the minimum viable slab depth is approached. However, the same trend is observed for both detailing cases which shows that the fact that the shallowest viable depth does not result in the least carbon intensive solution does not simply come from the detailing method. One assumption which can be made here is that this is due to the fact that a longer span is used.

It should also be noted that, as pointed out for the slabs using the two spans studied above, in the range of slab depths close to the optimal depth, differences in embodied carbon content are only of a few percents.

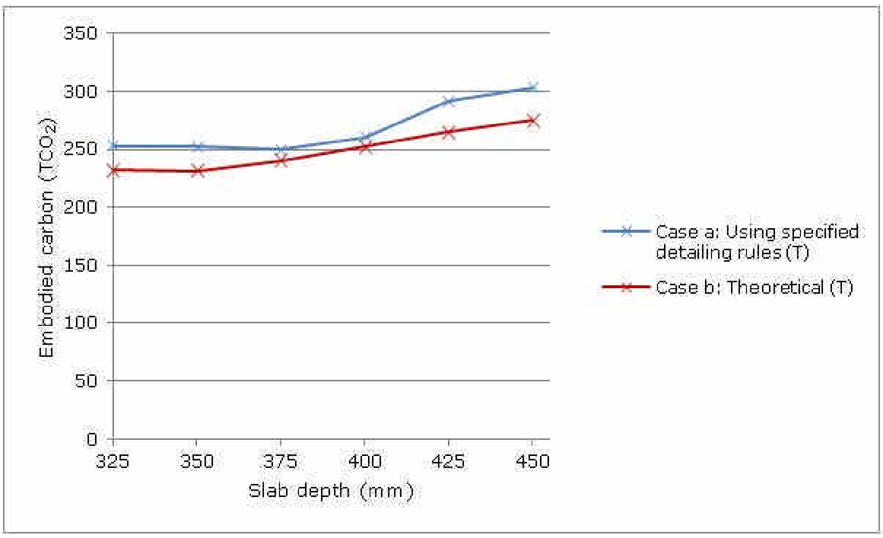


Figure 52: Embodied carbon as a function of the slab depth – 9.0mx9.0m column grid – 6x3 spans – C30/37

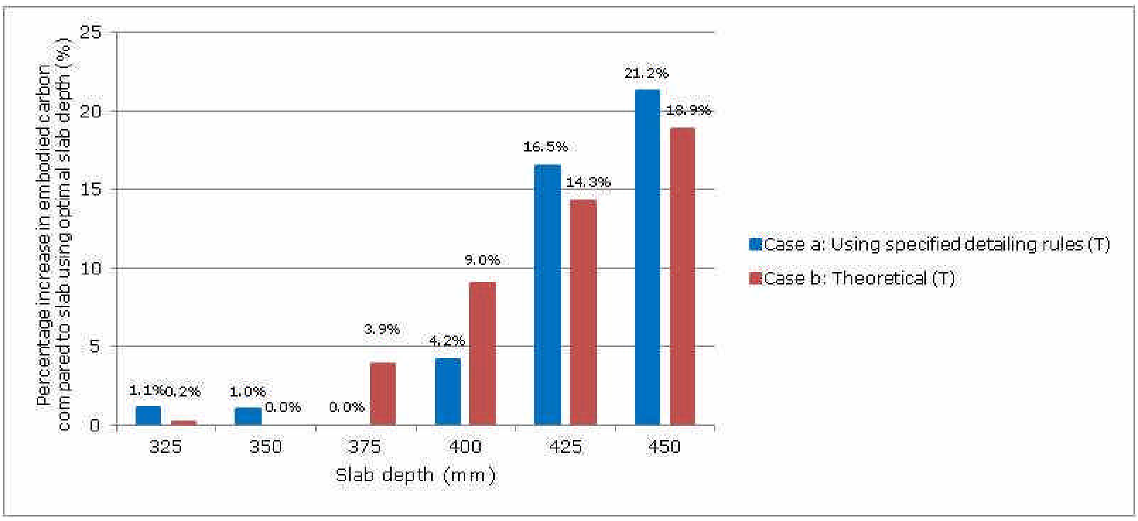


Figure 53: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth - 9.0mx9.0m column grid – 6x3 spans – C30/37

5.2.3.4 Punching shear

The results presented above do not account for punching shear. Some of the slabs considered are however quite shallow and need to be checked for punching shear. Moreover, the quantities of punching shear reinforcement may affect the results obtained above and need to be assessed.

5.2.3.4.1 Methodology

For the shallowest slab depth considered for each span, punching shear is checked at each of the three column types present in the slabs – inner, edge and corner – and the required punching shear reinforcement is calculated. This is done using a spreadsheet written by the author, which implements the requirements detailed in Section 6.4 of EN1992-1-1 (BSI 2004b).

5.2.3.4.2 Results

Figure I 1 to Figure I 9 in Appendix I present the punching shear checks and punching shear reinforcement provided. For the three spans considered, the shallowest slab depths tested are found to pass the punching shear checks of EN1992-1-1 provided that sufficient shear reinforcement is provided.

The total punching shear reinforcement for each of the shallowest slab depths tested for the three span dimensions considered is summarised in Table 15, and the quantities of longitudinal reinforcement and punching shear reinforcement are compared. In all three cases, the weight of punching shear reinforcement is found to represent a negligible proportion of the total weight of the longitudinal steel and can hence be considered to have a negligible effect on the results presented in the previous section.

5.2.3.5 Discussion

The study of the slabs presented above indicates that for each design situation there exists a slab depth which results in a least carbon intensive design. The results obtained highlight that thick slabs are more carbon intensive. The study of the quantities of reinforcement to be provided reveals that, above a certain thickness, thick slabs do not use less reinforcement than thinner ones, but at least as much and potentially more. This is due to the increase in the quantities of minimum reinforcement to be provided as the slab depth increases.
For the two smaller spans considered, 6.0m and 7.5m, the shallowest viable depth is in fact found to result in the least carbon intensive solution. For the 9.0m span, the optimal depth is found to be slightly larger than the minimum viable depth. These results are obtained in the case where the minimum quantities of reinforcement are provided as well as in the case where the actual bar sizes available are used. This seems to highlight that there is a certain span below which the optimal depth is the minimum viable depth and above which the optimal depth is larger than the minimum viable depth. More span dimensions could be tested to investigate this assumption, but this is beyond the scope of this study.

Large differences in embodied carbon are observed between slabs using relatively close depths. The study of the slabs supported on a 9.0mx9.0m column grid for example showed that a difference in embodied carbon content of as much as 16.5% could be obtained from a variation in slab depth of as little as 50mm. For all three spans however, variations in embodied carbon quantities for depths close to the optimal depth are found to be limited.

From this study, it would appear that potentially large savings are achievable by judiciously selecting the depth of a slab. And due to the fact that slabs using depths close to the optimal depth remain close to optimal, it seems possible to define for each span a slab depth or at least a range of slab depths to target to achieve minimum embodied carbon designs.

The results obtained in this section are based on the best estimates of the carbon intensities of concrete and rebar given in the ICE database (Hammond 2011). As previously highlighted, these numbers are subject to significant uncertainties. A question which emerges from these uncertainties is whether it is actually possible to make an informed choice regarding the depth to select to design a low embodied carbon slab, in the absence of precise data regarding the carbon intensities of the constituent materials. Indeed, at the design stage, the origins of the materials to be used for the construction of the building and their relative carbon intensities are not known. Does this compromise the ability of designers to make a decision on the depths to select to minimise the embodied carbon content of a slab? This question is the topic of the next section.

Span	Slab depth	Column type	Number of perimeters	Total number of links	Link diameter	Number of columns	Weight of punching shear reinforcement	Total weight of punching shear reinforcement	Total weight of longitudinal reinforcement	Percentage of weight of punching shear reinforcement in total reinforcement weight
(m)	(mm)	(-)	(-)	(-)	(mm)	(-)	(kg)	(kg)	(kg)	(%)
6.0	205	Inner	4	60	8	24	233	414	36800	1.1
		Edge	3	40	8	22	142			
		Corner	4	60	8	4	39			
7.5	260	Inner	6	90	8	12	222	525	40200	1.3
		Edge	6	70	8	16	230			
		Corner	7	90	8	4	74			
9.0	325	Inner	5	180	8	10	462	1113	71530	1.5
		Edge	5	130	8	14	467			
		Corner	6	180	8	4	185			

Table 15: Comparison of weight of punching shear reinforcement required in the shallowest slab depth tested for each span dimension and total weight of reinforcement provided in the slab

5.2.4 Sensitivity study: influence of uncertainties in embodied carbon intensities

As explained in Section 4.3.6, the best estimates of the carbon intensities of rebar and concrete given in the ICE database are only accurate to ±30%. A question which emerges from this uncertainty is whether it has the potential to modify significantly the slab depths for which the embodied carbon content of a slab in a given configuration is minimal.

To answer this question, two extreme scenarios are considered which account for these uncertainties. In the first scenario, a 30% lower embodied carbon intensity concrete and a 30% higher carbon intensity rebar, both relative to the best estimate values, are used. This scenario is denoted *GCBS* for '*Good Concrete Bad Steel*' in the rest of this section. The second scenario considers the opposite case: a 30% higher embodied carbon intensity concrete and a 30% lower carbon intensity rebar, denoted *BCGS* for '*Bad Concrete Good Steel*' in the following. These scenarios are extreme cases as they combine the two worst possible scenarios for each material. Although they are unlikely scenarios, they enable to fully assess the effect of these uncertainties.

The effects of these two scenarios on the results obtained in the previous section are presented below for the three spans considered.

5.2.4.1 6.0mx6.0m slab

Figure 54 presents, for the two extreme scenarios introduced above, the percentage increase in the embodied carbon content of a series of slabs using different depths compared to that of the slab using the depth resulting in the least carbon intensive design. The results are compared to those obtained in the case where the best estimates of the materials carbon intensities are used. On this Figure, the results are given for slabs detailed using the method described in Section 4.3.5.

Figure 55 presents the same results for slabs provided with the minimum reinforcement quantities theoretically required.

In both cases, in the *BCGS* scenario, thick slabs are found to be even more carbon intensive than in the case using the best estimates of the material carbon intensities. The depth of slab resulting in the least carbon intensive design is not affected by the change in carbon intensities and remains the shallowest viable slab depth.

In the *GCBS* scenario, very thick slabs are also found to be more carbon intensive than thinner ones. However, due to the higher carbon intensity of rebar relative to that of concrete, as the shallowest viable depth is approached, the increase in the quantities of rebar provided is such that the overall carbon content of the slab increases. In this scenario, the optimal slab depth is hence not found to be the shallowest viable one, 205mm, but a slightly thicker one at 225mm.

The results obtained in the previous section, where the best estimates of the carbon intensities are used, showed the two depths of 205mm and 225mm to result in slabs of very similar embodied carbon content. As highlighted on Figure 47, these only differ by a few percent.

Despite the significant magnitude of the uncertainties, the range of depths highlighted as resulting in the least carbon intensive designs using the best estimates of carbon intensities are still found to yield the best results in the two scenarios where uncertainties in materials carbon intensities are taken into account. Moreover, using either of these two depths in a scenario where it is not strictly optimal results in a limited departure from the strict optimal value:

- A slab designed using a depth of 205mm, which is optimal in the best estimate and *BCGS* scenarios, will only differ from the optimum by a maximum of 2% if the material carbon intensities turn out to be closer to those of the third scenario.
- Conversely, a slab designed using a depth of 225mm, which is optimal in the *GCBS* scenario, will only differ from the optimum by a maximum of 5.5% if the material carbon intensities turn out to be closer to those of the *BCGS* scenario.

These results enable us to conclude that, despite the significant uncertainties in the materials embodied carbon intensities, a range of slab depths resulting in low embodied carbon designs can be selected.

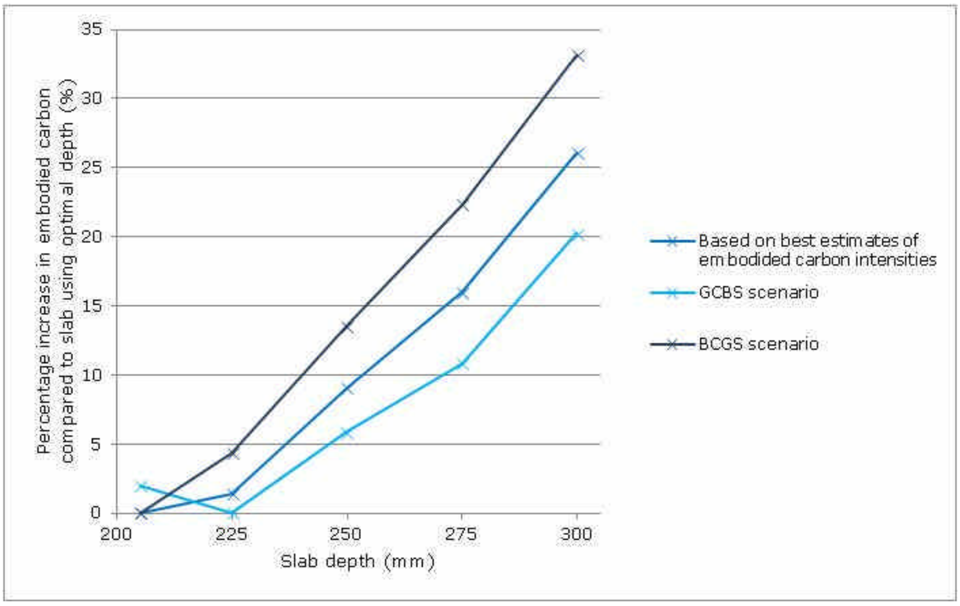


Figure 54: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 6.0mx6.0m column grid – 9x4 spans – C30/37 – Actual reinforcement

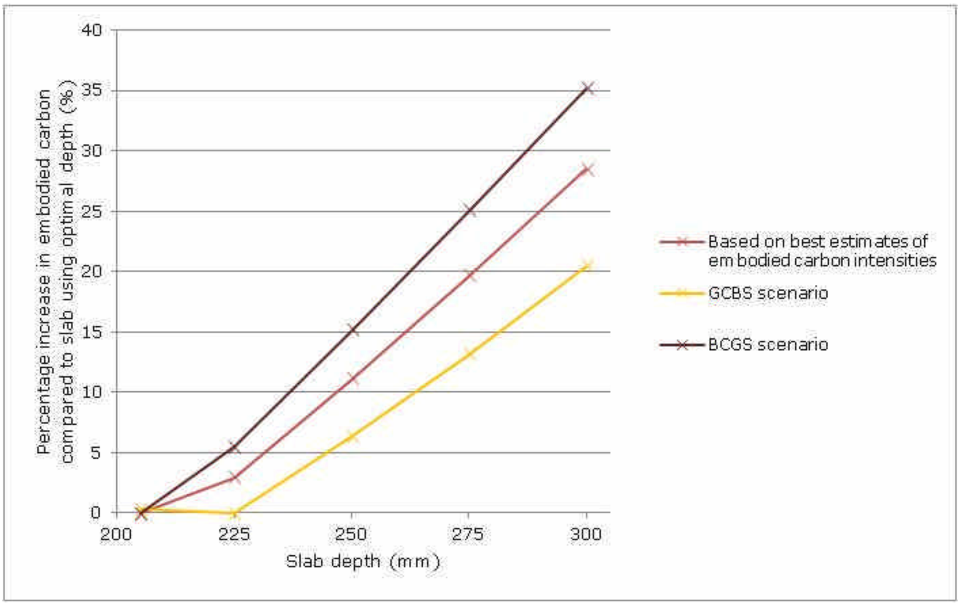


Figure 55: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 6.0mx6.0m column grid – 9x4 spans – C30/37 – Theoretical reinforcement

5.2.4.2 7.5mx7.5m slab

Figure 56 and Figure 57 present the same results for slabs supported on a 7.5mx7.5m column grid. The results are globally similar to those obtained for the slabs supported on a 6.0mx6.0m column grid and the same conclusions can be reached.

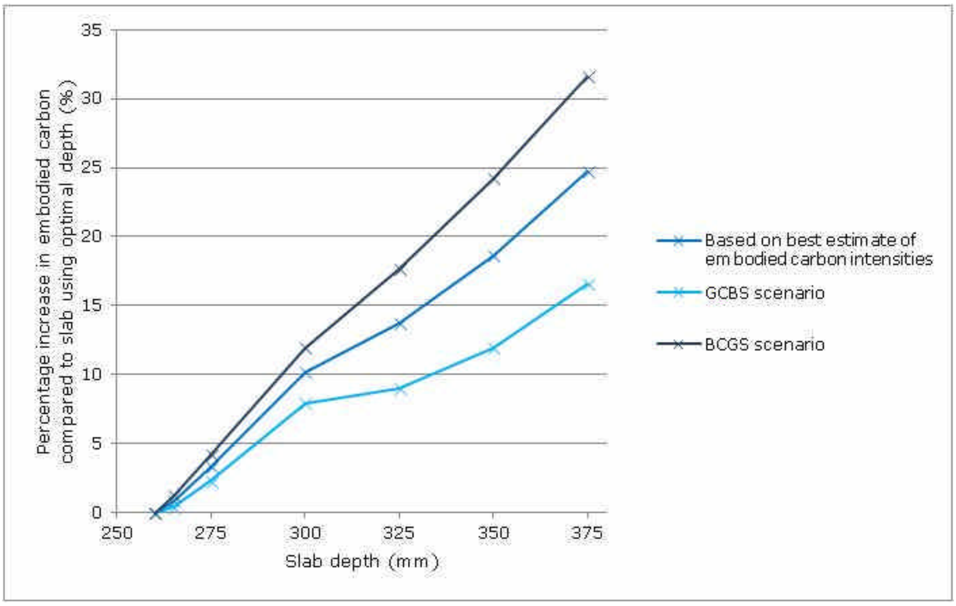


Figure 56: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 7.5mx7.5m column grid – 7x3 spans – C30/37 – Actual reinforcement

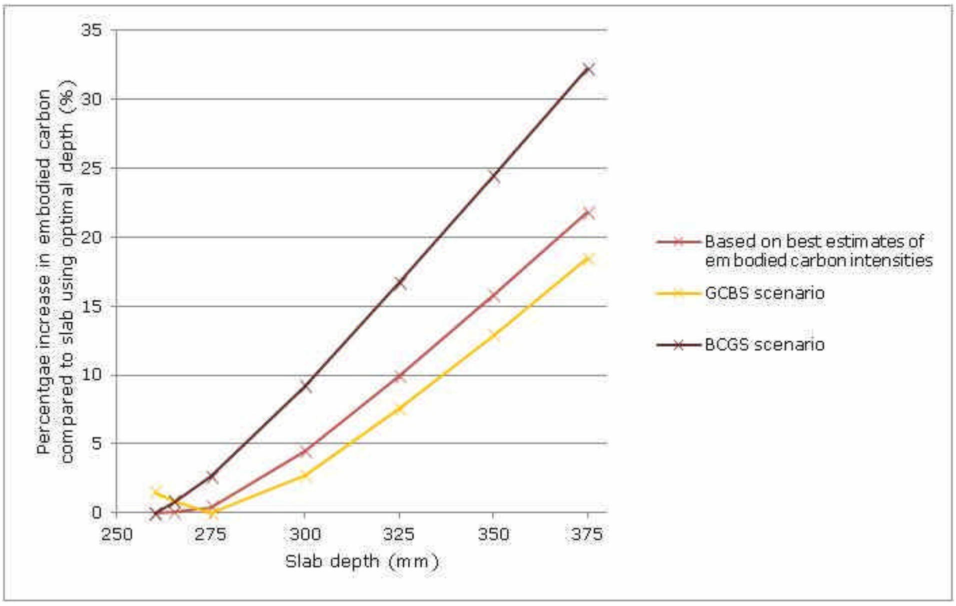


Figure 57: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 7.5mx7.5m column grid – 7x3 spans – C30/37 – Theoretical reinforcement

5.2.4.3 9.0mx9.0m slab

Figure 58 presents the same results for slabs supported on a 9.0mx9.0m column grid and detailed using the method described in Section 4.3.5.

In the scenario based on best estimates of the materials carbon intensities, the shallowest viable slab depth, 325mm, was previously shown not to be optimal, as a depth of 375mm was found to yield better results. The difference was however shown to be of only a few percent, and based on these carbon intensities, a slab using a depth between 325mm and 375mm was shown to be very close to optimal.

This trend of the shallowest viable slab not being optimal is exacerbated in the *GCBS* scenario as steel is penalised over concrete and shallower slabs tend to require greater quantities of steel. In this scenario, using a depth of 325mm results in a noticeably more carbon intensive slab than can be achieved by using the optimal depth which remains at 375mm.

The results obtained in the *BCGS* scenario reverse the trend and as concrete is penalised over steel, thinner slabs are found to be preferable. The shallowest viable slab depth, 325mm, is in fact found to result in the least embodied carbon design.

Here again, despite the significant magnitude of the uncertainty, a range of depths, 350mm to 375mm, can be selected which result in designs with an embodied carbon content close to optimal:

- A slab designed using a depth of 350mm only differs from the optimum by a maximum of 4.7% whichever scenario turns out to be the real one when materials are actually selected.
- A slab designed using a depth of 375mm, which is optimal in the *GCBS* scenario and when best estimates of the materials carbon intensities are used, will only differ from the optimum by a maximum of 4.6% if the material carbon intensities turn out to be closer to those of the *BCGS* scenario.

Figure 59 presents the same results for slabs supported on a 9.0mx9.0m column grid but in which only the minimum quantities of required reinforcement are provided. Results are similar except that the range of preferred depths is between 325mm and 350mm.

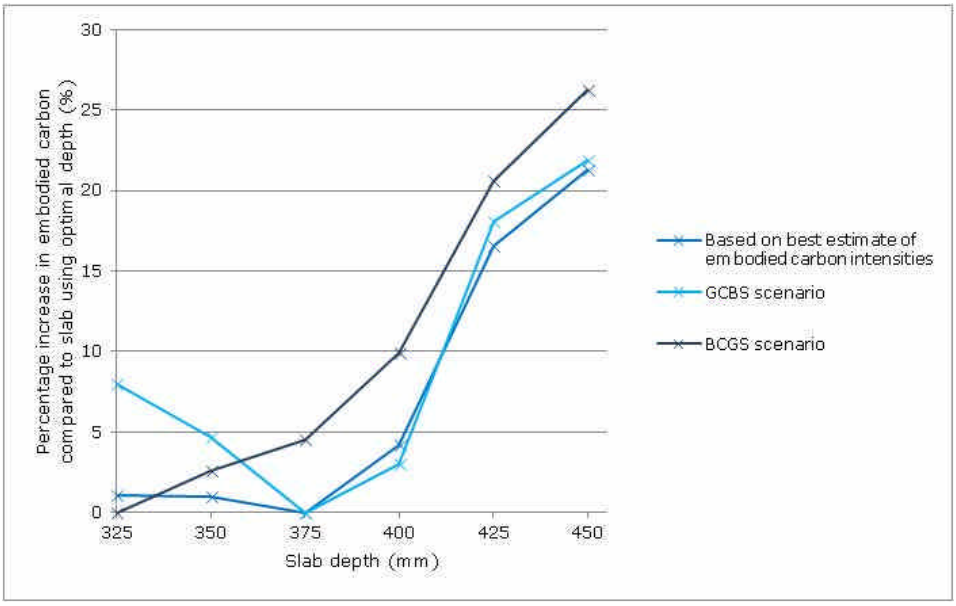


Figure 58: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 9.0mx9.0m column grid – 6x3 spans – C30/37 – Actual reinforcement

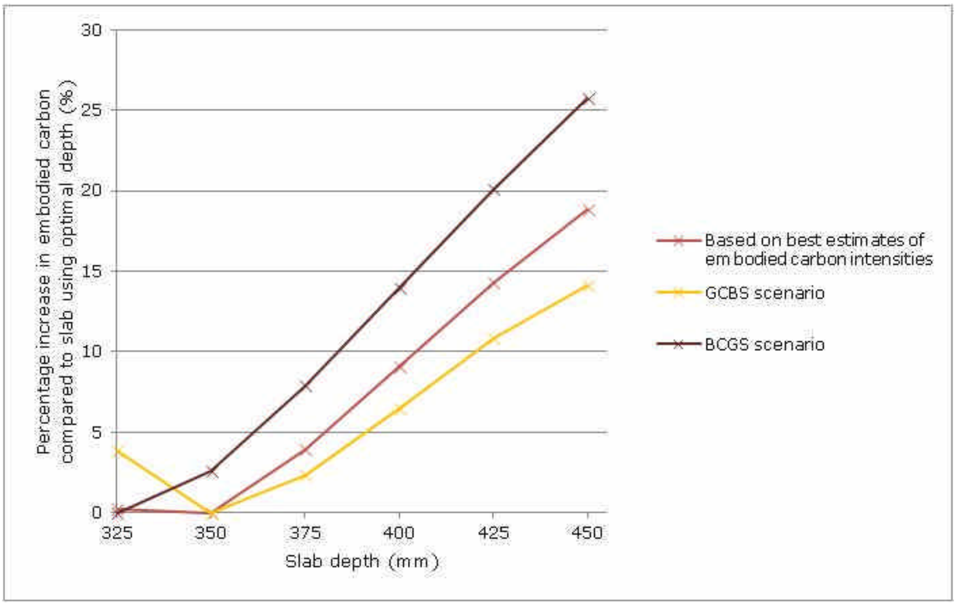


Figure 59: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 9.0mx9.0m column grid – 6x3 spans – C30/37 – Theoretical reinforcement

5.2.4.4 Discussion

By considering these two extreme scenarios to account for the uncertainties in the embodied carbon intensities of concrete and steel rebar, this study reveals that, despite the large values of these uncertainties, it is possible to make an informed choice regarding the selection of a slab depth to minimise the embodied carbon content of a slab. Despite the fact that uncertainties as high as 30% are considered in the carbon intensity of each of the two materials and that these may modify the global trends obtained, it is shown that it is possible to recommend a slab depth without compromising the optimality of the resulting slab by more than about 5%.

Consequently, it is possible to give guidance to designers in the form of span-to-effective-depth ratios to target in order to design slabs with minimum embodied carbon content. Table 16 summarises the span-to-effective-depth ratios found to result in less carbon intensive designs for the three spans considered in this first study when the detailing rules highlighted in Section 4.3.5 are used.

Span	Optimal slab depth	Cover	Largest bar diameter in bottom reinforcement	Effective depth	Span-to-effective depth ratio	Average span-to-effective-depth ratio
(m)	(mm)	(mm)	(mm)	(mm)	(-)	(-)
6.0	205	30	12	169	35.5	**33.5**
	225	30	10	190	31.6	
7.5	260	30	16	222	33.8	**32.7**
	275	30	16	237	31.6	
9.0	350	35	20	305	29.5	**28.3**
	375	35	16	332	27.1	

Table 16: Average span-to-effective depth ratios found to result in least embodied carbon design for each of three spans considered

5.2.5 Sensitivity study: influence of the size of a slab

As can be seen in Table H 1 to Table H 3 which describe the reinforcement provided in the slabs in detailing case a, as the depth of a slab approaches its minimum viable depth beyond which it fails in deflection, the reinforcement in the end spans and above the first interior supports needs to be increased significantly. This increase will have a greater effect on the embodied carbon content of slabs if the proportion of end spans in the total number of spans is larger, or in other words if the slab is smaller. The extreme case for the type of slabs considered in this study, which have at least three spans in each direction, is a slab with three spans in each direction. The question that emerges is whether, in this type of slabs, the optimal depth is significantly different from that for slabs of larger dimensions. To answer this question, the study carried out above is hence repeated on a series of three panels by three panels slabs.

5.2.5.1 Results

The results are presented in Figure 60 to Figure 65 for slabs supported on a grid of columns of respectively 6.0mx6.0m, 7.5mx7.5m and 9.0mx9.0m.

5.2.5.2 Discussion

For the two shortest spans of 6.0m and 7.5m, the reduced overall dimensions are found to have no significant influence of the slab depth or range of slab depths to target to achieve a minimum embodied carbon design.

In the case of the slabs supported on a 9.0mx9.0m column grid, the main effect is that the range of optimal depths is shifted slightly towards the larger depths, but the effect is limited.

This sensitivity study concludes this first investigation into the influence of the depth of a slab on its embodied carbon content.

The investigation has highlighted that for each design situation, there exists a slab depth at which the embodied carbon content of a slab is minimal. More importantly, it has shown that despite the large uncertainties which may exist in the carbon intensities of concrete and steel rebar at the time when a slab is designed, it is possible to make an informed choice regarding the slab depth or range of slab depths to select to produce a low embodied carbon design.

Guidance for designers could be given in the form of target span-to-effective-depth ratios. Different sets of target values could be produced for different design situations depending on the span, end usage, design loading etc. For the particular design situations tested, the size of the slab was found to have a limited influence on the span-to-depth ratios to target, but could be included as an additional parameter to increase the precision.

Guidance for designers could be given for different scenarios regarding the carbon intensities of the materials used in the slabs. In fact, three sets of target ratios could be given: one for the case where the relative carbon intensities of materials are unknown and built using the methodology described here with two extreme scenarios; one where concrete is expected to be better than average and steel is worse than average; and finally one for the opposite scenario.

In the following section, the influence of the concrete grade on the embodied carbon content of flat slabs is investigated.

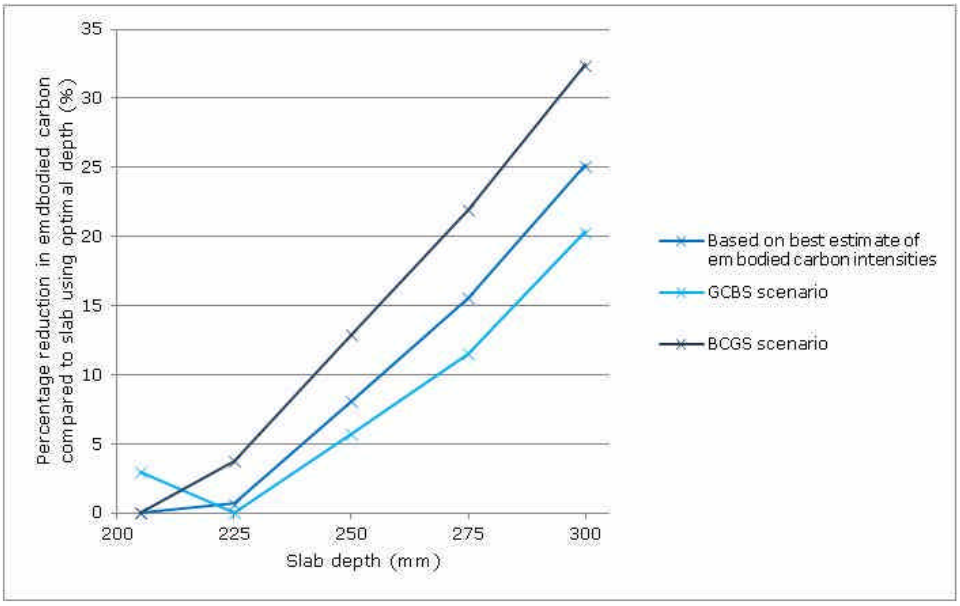


Figure 60: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 6.0mx6.0m column grid – 3x3 spans – C30/37 – Actual reinforcement

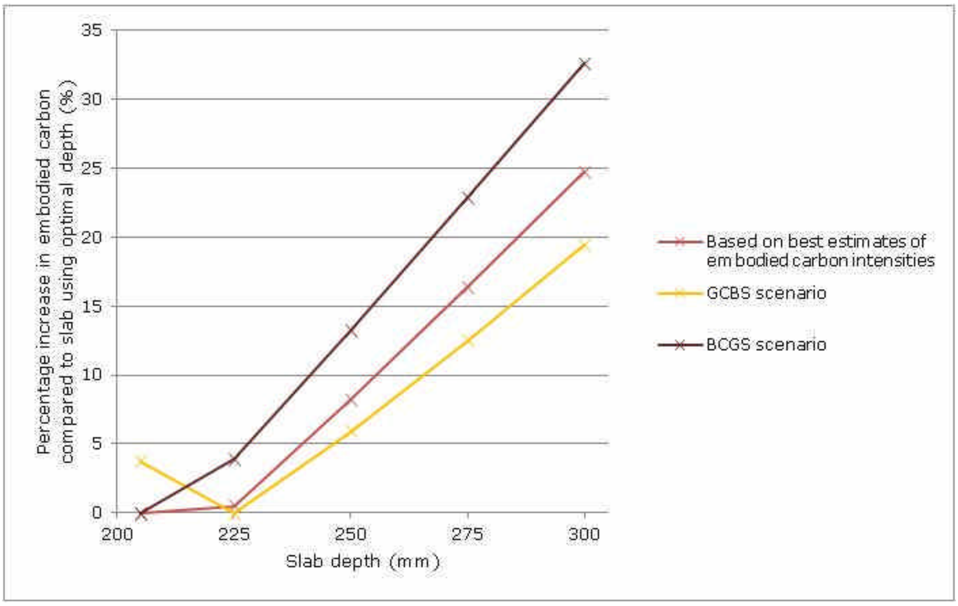


Figure 61: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 6.0mx6.0m column grid – 3x3 spans – C30/37 – Theoretical reinforcement

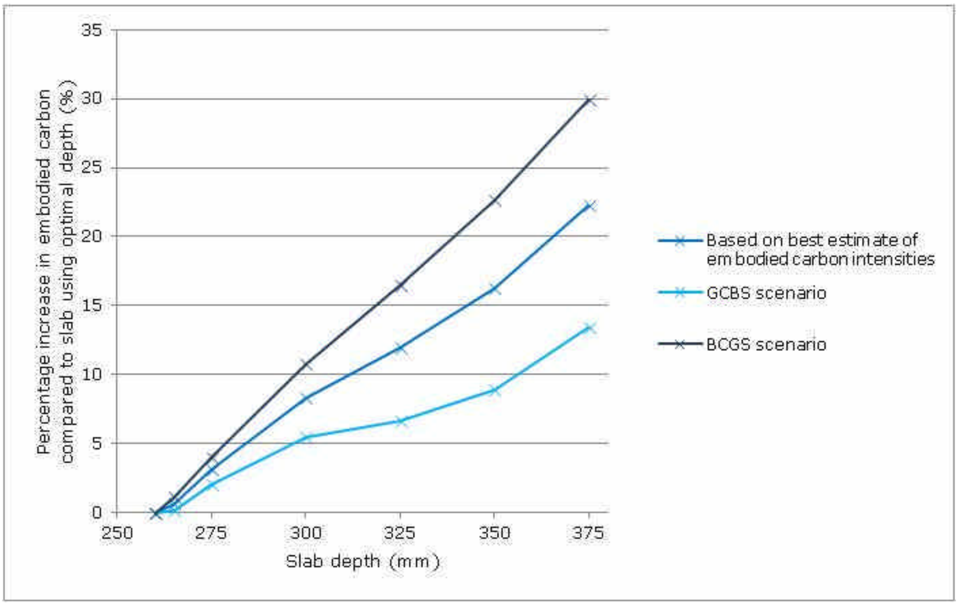


Figure 62: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 7.5mx7.5m column grid – 3x3 spans – C30/37 – Actual reinforcement

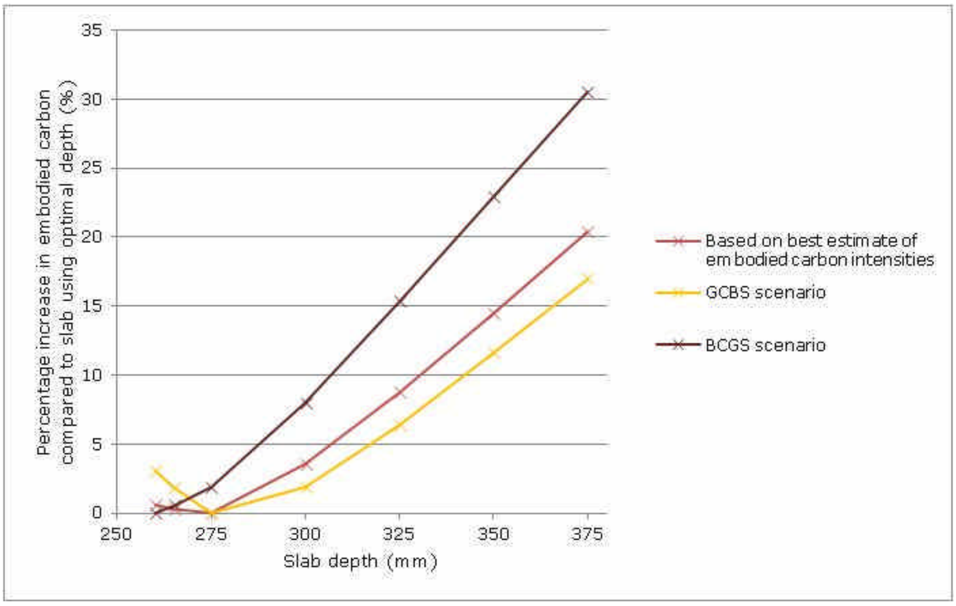


Figure 63: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 7.5mx7.5m column grid – 7x3 spans – C30/37 – Theoretical reinforcement

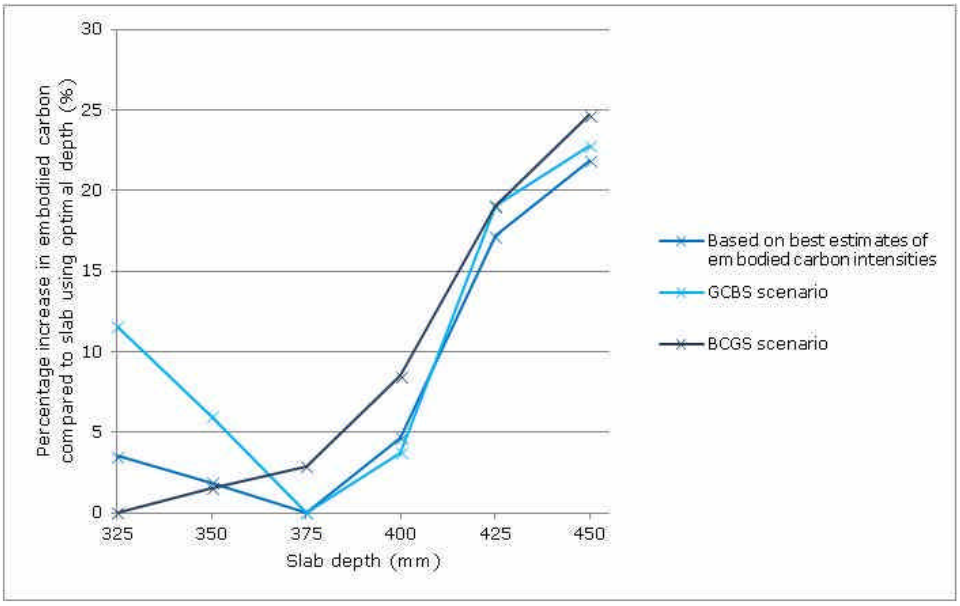


Figure 64: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 9.0mx9.0m column grid – 3x3 spans – C30/37 – Actual reinforcement

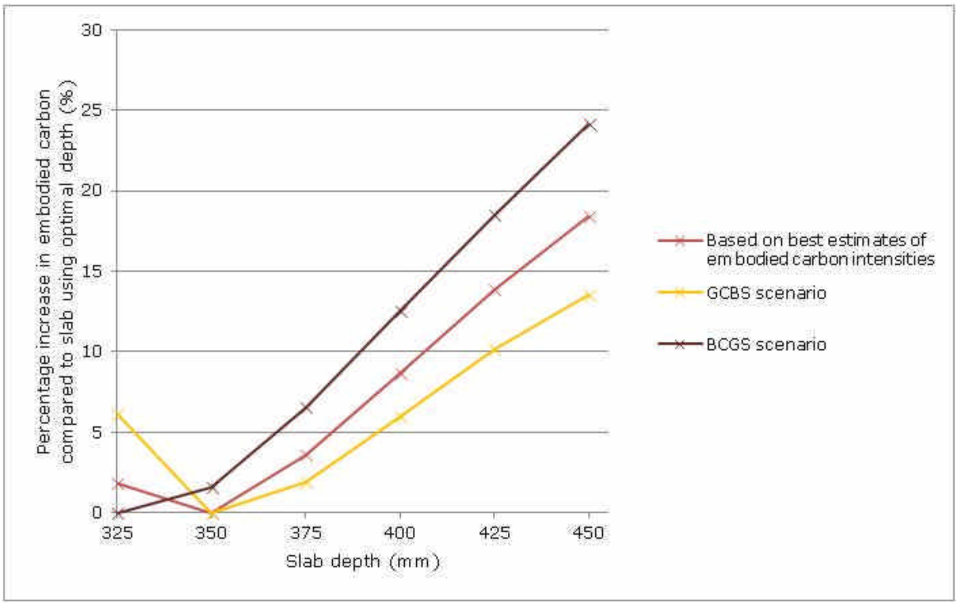


Figure 65: Percentage increase in embodied carbon compared to embodied carbon in slab using optimal depth – 9.0mx9.0m column grid – 3x3 spans – C30/37 – Theoretical reinforcement

5.3 INFLUENCE OF CONCRETE GRADE ON EMBODIED CARBON CONTENT OF SLABS

Using a higher grade of concrete in the design of a flat slab potentially enables the use of shallower slabs due to the increased concrete modulus of elasticity, and may result in reduced quantities of reinforcement from the increased concrete strength. However, as made obvious in Table 12 which summarises the carbon intensities of different grades of concrete, higher grades of concrete also have larger embodied carbon intensities due to the additional quantities of cement required.

This section presents an investigation into the influence of the choice of concrete grade on the embodied carbon of flat slabs.

5.3.1 Characteristics of slabs

This section presents the characteristics of the slabs considered for investigation. It covers the spans and the range of slab depths considered for each span, the grade of concrete used and the size of the slab considered for each span.

For this part of the study, a span of 7.5m is used for investigation. A single value is considered here as it is expected that the trends obtained for one particular span hold true for other spans.

The slab depths considered are as per the principle described in Section 5.2.

Three concrete grades are considered in addition to the C30/37 grade considered in the previous part of the study: one lower grade of C28/35, and two higher grades of respectively C35/40 and C40/50.

A 7x3 spans slab is considered, as it approaches the dimensions of a 50mx25m building which is considered typical of contemporary residential buildings. This should however have no impact on the results obtained here.

5.3.2 Reinforcement provided

The methodology previously described in Section 4.3 is used to work out the reinforcement to be provided in the slabs. The reinforcement provided in each slab using the reinforcement detailing rules described in Section 4.3.5 is presented in Table H 4 to Table H 6 in Appendix H, for concrete grades of respectively C28/35, C35/45 and C40/50.

The results are also discussed for slabs using the quantities of reinforcement theoretically required. This is done in order to get an understanding of whether the results obtained are dependent on the fact that a finite number of bar sizes and spacings are used.

5.3.3 Results

The results are presented below, first for slabs detailed using the detailing method described in Section 4.3.5 and then for slabs using the quantities of reinforcement theoretically required.

5.3.3.1 Using detailing method described in Section 4.3.5

The results obtained are presented in Table 17 for the slabs detailed using the method described in Section 4.3.5. Slabs using higher grades of concrete are constantly found to be more carbon intensive than those using lower grades, even when using shallower depths than can be achieved using lower concrete grades.

5.3.3.2 Based on theoretical reinforcement required

The results obtained are presented in Table 18 for the slabs detailed using the minimum quantities of reinforcement required. As above, slabs using higher concrete grades are found to be more carbon intensive.

5.3.4 Interpretation

The initial reason to investigate the influence of the grade of concrete used on the embodied carbon content of slabs was that although higher grade concretes are more carbon intensive, a slab using a higher concrete grade would be likely to require less reinforcement.

Table 19 and Table 20 present the quantities of reinforcement provided in the slabs for the four concrete grades considered, using respectively the detailing method presented in Section 4.3.5 and the theoretical quantities of reinforcement required.

In the case where the minimum quantities of reinforcement required are provided, slabs using lower grades of concrete are in fact found to require less reinforcement than those using higher concrete grades for most depths apart from the shallowest. Table 22 presents the minimum quantities of reinforcement required at each point in a 260mm thick slab using different concrete grades. Although as a higher grade is used the quantities of reinforcement required at each of the eight points highlighted on Figure 41 reduce, a greater amount of minimum reinforcement is also required. This results in limited overall savings in reinforcement quantities and is not sufficient to counterbalance the increased carbon intensity of the concrete.

In slabs where the reinforcement detailing rules presented in Section 4.3.5 are used, the quantities of reinforcement in slabs using lower concrete grades are constantly found to be lower than in slabs using higher ones. As can be seen from Table 21, which summarises the quantities of reinforcement provided in 260mm thick slabs using the four grades of concrete considered here, this is due to the fact that standard bar sizes are used and in the case of the 260mm thick slabs for example, the provided bar sizes do not actually change as the concrete grade varies. Added to the fact that a greater area of minimum reinforcement needs to be provided as the concrete grade increases, using higher concrete grades results in more carbon intensive designs.

5.3.5 Discussion

The results presented above show that, as far as the slabs are concerned, using higher concrete grades results in more carbon intensive designs. But the results also show that higher concrete grades permit the use of shallower slabs which at the scale of the whole building may be beneficial on the total embodied carbon quantities, through a reduction in the loads on the supporting structure: columns and foundations.

To test whether this impact is likely to be significant, the reduction in load obtainable by using the shallowest slab viable with a C40/50 concrete grade, instead of that using a C28/35 concrete grade, is assessed.

The percentage reduction in the total load will be greater in cases where the imposed load is smallest. For multi-storey buildings of more than two storeys, the total imposed load seen by columns may be reduced as per Clause 6.2.2(2) of EN1991-1-1 (BSI 2002) by the following factor:

$$\alpha_n = \frac{2 + (n-2) \times \psi_0}{n}$$

where $n \geq 2$ is the number of storeys and $\psi_0 = 0.7$ as per Table NA.A1.1 of the National Annex to EN1990 (BSI 2004).
The lowest value possible for α_n is 0.7, and this value is used in the following calculations.

ULS surface load from 260mm thick slab:

$$1.35 \times (SW + SDL) + 1.5 \times IL = 1.35 \times (260 \times 25 \times 10^{-3} + 3) + 1.5 \times 1.5 \times 0.7 = 14.40kN/m^2$$

ULS surface load from 250mm thick slab:

$$1.35 \times (SW + SDL) + 1.5 \times IL = 1.35 \times (250 \times 25 \times 10^{-3} + 3) + 1.5 \times 1.5 \times 0.7 = 14.06kN/m^2$$

A reduction in slab depth from 260mm to 250mm hence results in a reduction in the ULS design surface loads from the slabs of less than 2.5%. Compared to the increase in embodied carbon of 19.7% between a 260mm thick slab in C28/35 and a 250mm thick slab in C40/50, this reduction is negligible.

This short calculation confirms that slabs using higher concrete grades result in more carbon intensive designs.

This concludes this second investigation into the influence of the choice of concrete grade on the embodied carbon content of flat slabs.

Slab depth	Embodied carbon in slab			
	C28/35	C30/37	C35/45	C40/50
(mm)	(TCO_2)	(TCO2)	(TCO2)	(TCO2)
375	187.2	194.0	224.4	238.0
350	178.1	184.5	198.2	227.4
325	170.9	176.8	189.6	201.4
300	158.2	171.3	180.4	191.3
275	149.1	160.7	172.6	185.9
260	150.2	155.5	166.4	179.0
255	-	-	171.2	179.3
250	-	-	-	179.8

Table 17: Embodied carbon in slabs using reinforcement detailed using detailing method of Section 4.3.5 as a function of the slab depth for different grades of concrete – 7.5mx7.5m column grid – 7x3 spans

Slab depth	Embodied carbon in slab			
	C28/35	C30/37	C35/45	C40/50
(mm)	(TCO_2)	(TCO2)	(TCO2)	(TCO2)
375	176.7	184.3	201.3	217.2
350	167.3	174.3	189.4	204.5
325	158.1	164.6	178.9	192.3
300	149.4	155.4	168.3	180.7
275	142.0	148.3	158.3	169.4
260	143.3	147.1	155.4	162.9
255	-	-	157.8	162.1
250	-	-	-	161.2

Table 18: Embodied carbon in slabs based on theoretical reinforcement as a function of the slab depth for different grades of concrete – 7.5mx7.5m column grid – 7x3 spans

Slab depth	Quantities of reinforcement provided			
	C28/35	C30/37	C35/45	C40/50
(mm)	**(T)**	**(T)**	**(T)**	**(T)**
375	36.4	36.4	47.5	47.5
350	36.4	36.4	36.4	48.1
325	37.7	37.7	37.7	37.7
300	32.1	40.6	38.6	38.6
275	35.1	39.8	40.6	42.9
260	39.8	40.2	40.6	42.9
255	-	-	45.6	46.8
250	-	-	-	

Table 19: Embodied carbon in slabs using reinforcement detailed using detailing method of Section 4.3.5 as a function of the slab depth for different grades of concrete – 7.5mx7.5m column grid – 7x3 spans

Slab depth	Quantities of reinforcement provided			
	C28/35	C30/37	C35/45	C40/50
(mm)	**(T)**	**(T)**	**(T)**	**(T)**
375	28.9	29.4	31.0	32.7
350	28.6	29.1	30.0	31.8
325	28.5	29.0	30.1	31.2
300	25.8	29.2	30.0	31.1
275	30.0	30.9	30.4	31.2
260	34.8	33.4	32.8	31.4
255	-	-	36.0	33.5
250	-	-	-	

Table 20: Embodied carbon in slabs based on theoretical reinforcement as a function of the slab depth for different grades of concrete – 7.5mx7.5m column grid – 7x3 spans

Concrete grade	Basic mesh	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
		Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
C28/35	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
C30/37	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
C35/45	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
C40/50	H12@200	H20@200	H16@200	H10@200	H10@200	H12@200	H10@200

Table 21: Provided reinforcement in 260mm thick slab as a function of concrete grade using detailing rules of section 4.3.5 – 7.5mx7.5m column grid – 7x3 spans

Concrete grade	Minimum area of reinforcement required	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
		Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
(-)	(mm^2/m)	(mm^2/m)	(mm^2/m)	(mm^2/m)	(mm^2/m)	(mm^2/m)	(mm^2/m)
C28/35	324	1855	1303	431	315	1364	681
C30/37	339	1840	1296	431	315	1285	681
C35/45	376	1812	1283	431	315	1109	681
C40/50	409	1793	1280	433	317	916	671

Table 22: Theoretical reinforcement required in 260mm thick slab as a function of concrete grade – 7.5mx7.5m column grid – 7x3 spans

5.4 PUTTING RESULTS IN CONTEXT

In the two previous sections, investigations have been presented into the impact of the selection of a slab depth and of the concrete grade on the embodied carbon content of flat slabs, and recommendations have been made to reduce it.

This section puts the savings highlighted in the context of practical design situations. To do so, the reductions in the carbon content of flat slabs achievable by using cement replacement products are assessed. Using cement replacement products is currently the most popular design measure when it comes to attempts to reduce the carbon content, and more generally the embodied environmental impact, of reinforced concrete structures. These savings give a point of comparison to assess the impact of the measures investigated in this thesis. The impact of the choice of concrete grade can be immediately compared and discussed. To assess the impact of the choice of the slab depth, two design scenarios are considered, which would result in another depth than the optimal one to be selected, and the results are used to assess the importance of the choice of the depth of a slab.

5.4.1 Influence of using cement replacement products on embodied carbon content of slabs

In this section, the influence of using cement replacement products on the embodied carbon content of slabs is assessed. A methodology is first proposed, followed by a presentation of the results

5.4.1.1 Methodology

The Concrete Centre (2011) gives guidance on how to specify concrete with cement replacement products. Their guidance indicates that the most commonly used proportion of GGBS and fly ash are respectively of 50% and 25% by mass of the total cementitious content. These proportions are used here.

The ICE database gives carbon intensities for concretes of grades C28/35 and C32/40 and using cement replacement products. These are summarised in Table 23.

For the two concrete grades considered, C28/35 and C32/40, using 25% fly ash results in a reduction of 18.9% and 19.6% respectively in the carbon intensity of concrete, while using 50% GGBS results in reductions of 40.5% and 38.6% respectively. The values for C30/37 concrete are obtained by linear interpolation. The following reductions are considered:

- Using 25% fly ash is considered to result in a 19.2% reduction in the carbon intensity of concrete, and
- Using 50% GGBS is considered to result in a 39.5% reduction in the carbon intensity of concrete.

Concrete type	Embodied carbon
	(kgCO2/kg)
C28/35 without replacement products	0.148
C28/35 with 25% fly ash	0.120
C28/35 with 50% GGBS	0.088
C32/40 without replacement products	0.163
C32/40 with 25% fly ash	0.131
C32/40 with 50% GGBS	0.100

Table 23: Carbon intensities of concrete with cement replacement products

5.4.1.2 Results

Table 24 and Table 25 present the reduction in embodied carbon content obtained for slabs supported on a 7.5mx7.5m column grid, when the detailing rules specified in Section 4.3.5 are used.

The savings achievable by using a concrete with 25% fly ash and a concrete with 50% GGBS are found to be respectively around 13% and 27%.

5.4.2 Impact of selecting a higher concrete grade

Regarding the impact of using a higher concrete grade, Table 26 summarises the reductions in embodied carbon content achievable by using a C28/35 concrete grade instead of higher grades. The reductions obtained by using a C28/35 concrete instead of C30/37 range between 3% and 8% depending on the depth of the slab. Those obtainable by using a C28/35 concrete instead of C35/45 range between 10% and 16%. The reductions obtainable by using lower concrete grades are hence of a similar order as those achievable by using cement replacement products. The increase in embodied carbon associated with the use of higher grades of concrete is hence non-negligible and using higher concrete grades should hence be avoided.

5.4.3 Impact of selecting a non-optimal slab depth

Regarding the impact of the selection of the depth of a slab on its embodied carbon content, two scenarios are considered which would result in the selection of a depth other than the one resulting in the least carbon intensive design.

Scenario 1

The building used as a case-study in the previous study by the author (Thirion 2010) uses flat slabs of 3x8 bays. The plan arrangement of the building, given in Figure 66, is typical of contemporary residential design and construction: two cores are located at the two ends of the building, which house the stairs and lifts to provide access to the upper floor. Due to the location of the cores in the middle bay in the short direction, the slab bays adjacent to the cores, highlighted in yellow in Figure 66, are simply-supported. This means that a greater depth than the minimum depth viable for a slab with three continuous spans is required in these spans for the slab not to fail in deflection.

The effect of using a depth based on these simply-supported panels in the whole slab is investigated. A slab supported on a 7.5mx7.5m grid is selected as this is the closest of the three spans studied previously to the dimensions of the actual building. Designing these four bays as simply-supported results in a slab depth of 325mm for the slab not to fail in deflection. Introducing a step in the slab for these four bays to be thicker while the rest of the slab uses the optimal depth obtained in Section 5.2 would result in a reduction in the embodied carbon in the slab of between 8% and 15% depending on the actual carbon intensity of the materials.

Scenario 2

Placing shear links is a time consuming activity and generally, the number of shear link perimeters is sought to be kept in the lower ranges. In the building used as a case-study in the previous study by the author (Thirion 2010) the maximum number of shear link perimeters provided is three.

The slab depth required to keep the number of shear link perimeter at three is investigated for the case of the 7.5mx7.5m slab studied previously. For the number of perimeters to become three above the inner columns, the thickness of the slab has to be increased from 260mm to 330mm, as detailed in Figure I 10. From the results of Figure 50, this change in the design would result in an increase in embodied carbon content of around 14%.

In the two scenarios considered, the effect of not using the optimal slab depth is found to have a similar effect as not using cement replacement products. This highlights that adequately selecting the depth of a slab may result in non-negligible savings in embodied carbon.

This section concludes the main part of the study into the influence of design choices on the embodied carbon content of flat slabs.

In the following section, the impact of detailing using a fixed bar spacing, and a finite number of standardised bar sizes, on the quantities of steel rebar provided in the slab and on the quantities of carbon embodied in the slabs, is investigated.

Slab depth	Basic case: without cement replacement products	With 25% fly ash	
	Embodied carbon in slab	Embodied carbon in slab	Reduction compared to basic case
(mm)	(kgCO2/kg)	(kgCO2/kg)	(%)
375	194.0	166.5	14.2
350	184.5	158.8	13.9
325	176.8	153.0	13.5
300	171.3	149.3	12.8
275	160.7	140.5	12.5
265	156.9	137.4	12.4
260	155.5	136.5	12.2

Table 24: Reduction in embodied carbon from using a cement with 25% fly ash - 7.5x7.5 column grid – 7x3 spans – C30/37

Slab depth	Basic case: without cement replacement products	With 50% GGBS	
	Embodied carbon in slab	Embodied carbon in slab	Reduction compared to basic case
(mm)	(kgCO2/kg)	(kgCO2/kg)	(%)
375	194.0	137.5	29.1
350	184.5	131.7	28.6
325	176.8	127.8	27.7
300	171.3	126.1	26.4
275	160.7	119.2	25.8
265	156.9	116.9	25.5
260	155.5	116.3	25.2

Table 25: 7.5x7.5: Reduction in embodied carbon from using a cement with 50% GGBS - 7.5x7.5 column grid – 7x3 spans – C30/37

Slab depth	Percentage reduction obtained by using C28/35 instead of C30/37	Percentage reduction obtained by using C28/35 instead of C35/45
(mm)	(%)	(%)
375	3.5	16.6
350	3.5	10.1
325	3.3	9.9
300	7.6	12.3
275	7.2	13.6
260	3.4	9.7

Table 26: Reductions obtainable by using C28/35 concrete grade instead of higher grades

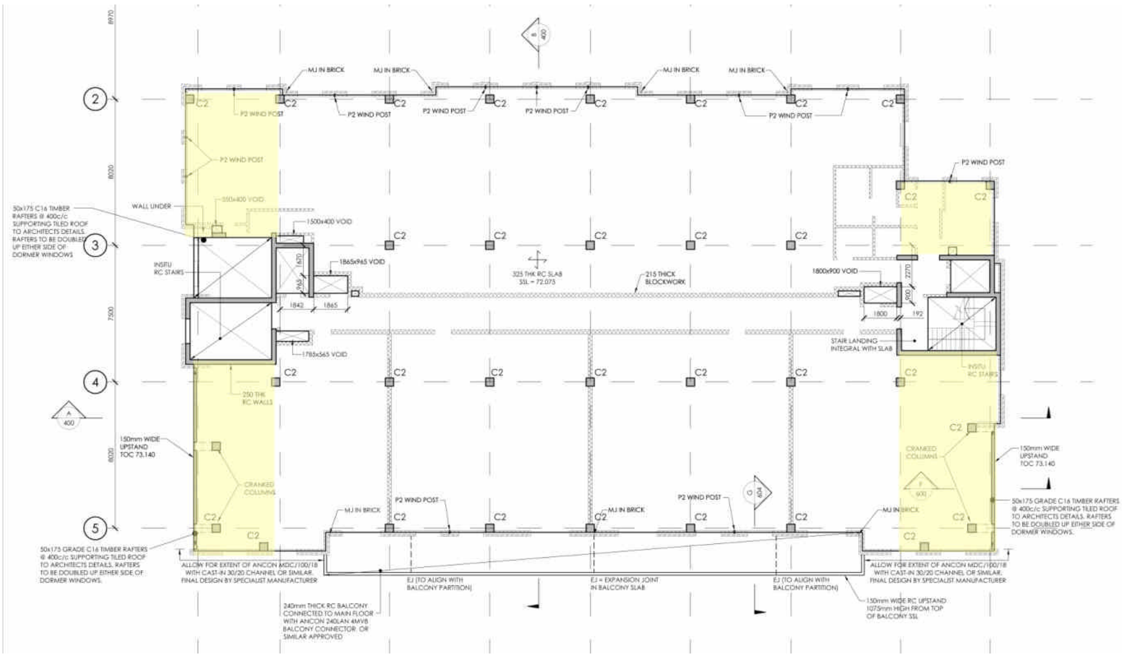


Figure 66: Plan view of building used as case study in previous study (Thirion 2010) with simply-supported bays adjacent to the cores highlighted

5.5 INFLUENCE OF USING A STANDARDISED FINITE SET OF BAR SIZES AND SPACINGS

Although this aspect is not one of the main areas of investigation of this study, the results obtained during the investigation of the influence of the depth of a slab on its embodied carbon content create an opportunity to discuss the influence on embodied carbon quantities of using a standardised finite set of bar sizes and a finite number of bar spacings in the detailing of the reinforcement to be provided in a slab.

Indeed, in this previous study, in order to best explain the results obtained concerning the reinforcement quantities to be provided using the detailing method described in Section 4.3.5, the minimum theoretical quantities of reinforcement to be provided if an infinite number of bar sizes and spacings could be used were calculated. Figure 45, Figure 48 and Figure 51 present these two quantities of reinforcement as a function of the slab depth for spans of respectively 6.0m, 7.5m and 9.0m. From these figures, the gap between the two quantities of reinforcement can be seen to be significant.

In this section, the savings achievable if a reinforcement exactly matching the theoretical reinforcement required could be provided are assessed and discussed. This discussion falls beyond the area of investigation proposed for this study which is a review of current design practice, and is more related to the area of investigation of the study presented in Chapters 2 and 3 as it requires an advance in technology.

5.5.1 Slabs characteristics and reinforcement provided

For this investigation, the slabs used in Section 0 for the study of the influence of the slab depth on the carbon intensities of slabs are re-used. The same concrete grade of C30/37 is also considered. The reinforcement provided is hence also similar.

5.5.2 Results

Table 27, Table 29 and Table 31 present a comparison between the quantities of reinforcement to be provided using a finite set of bar sizes and those strictly required theoretically. For the shallower slab depths previously highlighted as resulting in a lower embodied carbon content, differences of between 10% and 30% are obtained between the two quantities of rebar.

Table 28, Table 30 and Table 32 present the same results in terms of total embodied carbon in the slabs. For the same range of depths, savings of between 4% and 11% are obtained.

5.5.3 Discussion

Although they are theoretical results as a certain degree of rationalisation will always exist, the savings in embodied carbon, and even more, in steel rebar quantities are non-negligible.

Bamtec Ltd has developed an automated process to manufacture steel mats which use reinforcing bars of varying diameters, varying lengths and at varying centres to closely match the amount of steel required at each point in a slab (Bamtec 2012). The mats are delivered to site and rolled out. The savings in steel rebar claimed by the manufacturer are of a similar order of magnitude as obtained here. Interviews with designers pointed out that the specification of such a technology is beyond their control as they are required to produce a design buildable by any contractor and this technology may bear a cost premium. The choice to use such a technology is hence the contractor’s. In view of the obtainable savings, the use of such a technology in building projects should be encouraged.

Slab depth	Weight of rebar		
	Using finite set of bar sizes	Providing theoretical reinforcement required	Reduction in embodied carbon
(mm)	(T)	(T)	(%)
300	33.9	23.7	30.1
275	31.6	23.3	26.2
250	32.2	23.2	28.0
225	32.2	23.5	27.0
205	36.8	26.9	26.9

Table 27: Comparison between weights of steel reinforcement required using finite set of bar sizes and spacings and theoretical reinforcement required for a range of slab depth – 6.0mx6.0m column grid– 9x4 spans – C30/37

Slab depth	Embodied carbon in slab		
	Using finite set of bar sizes	Providing theoretical reinforcement required	Reduction in embodied carbon
(mm)	(TCO_2)	(TCO_2)	(%)
300	173.0	158.5	8.4
275	159.2	147.6	7.3
250	149.6	137.1	8.4
225	139.2	127.0	8.8
205	137.2	123.3	10.1

Table 28: Comparison between embodied carbon quantities in slabs with steel reinforcement required using finite set of bar sizes and spacings and theoretical reinforcement required for a range of slab depth – 6.0mx6.0m column grid– 9x4 spans – C30/37

Slab depth	Weight of rebar		
	Using finite set of bar sizes	Providing theoretical reinforcement required	Reduction in embodied carbon
(mm)	(T)	(T)	(%)
375	36.4	29.4	19.2
350	36.4	29.1	20.1
325	37.7	29.0	23.1
300	40.6	29.2	28.1
275	39.8	30.9	22.4
265	39.8	33.0	17.1
260	40.2	34.2	14.9

Table 29: Comparison between weights of steel reinforcement required using finite set of bar sizes and spacings and theoretical reinforcement required for a range of slab depth – 7.5mx7.5m column grid– 7x3 spans – C30/37

Slab depth	Embodied carbon in slab		
	Using finite set of bar sizes	Providing theoretical reinforcement required	Reduction in embodied carbon
(mm)	(TCO_2)	(TCO_2)	(%)
375	194.0	184.3	5.0
350	184.5	174.3	5.5
325	176.8	164.6	6.9
300	171.3	155.4	9.3
275	160.7	148.3	7.7
265	156.9	147.3	6.1
260	155.5	147.1	5.4

Table 30: Comparison between embodied carbon quantities in slabs with steel reinforcement required using finite set of bar sizes and spacings and theoretical reinforcement required for a range of slab depth – 7.5mx7.5m column grid– 7x3 spans – C30/37

Slab depth	Weight of rebar		
	Using finite set of bar sizes and spacings	Providing theoretical reinforcement required	Reduction in embodied carbon
(mm)	(T)	(T)	(%)
450	65.6	45.8	30.3
425	65.6	45.7	30.5
400	52.0	45.1	13.3
375	52.9	45.1	14.8
350	63.0	47.0	25.3
325	63.3	55.9	11.8

Table 31: Comparison between weights of steel reinforcement required using finite set of bar sizes and spacings and theoretical reinforcement required for a range of slab depth – 9.0mx9.0m column grid– 6x3 spans – C30/37

Slab depth	Embodied carbon in slab		
	Using finite set of bar sizes	Providing theoretical reinforcement required	Reduction in embodied carbon
(mm)	(TCO_2)	(TCO_2)	(%)
375	303.2	275.2	9.2
350	291.4	264.6	9.2
325	260.6	252.4	3.1
300	250.1	240.6	3.8
275	252.5	231.5	8.3
265	241.3	232.1	3.8

Table 32: Comparison between embodied carbon quantities in slabs with steel reinforcement required using finite set of bar sizes and spacings and theoretical reinforcement required for a range of slab depth – 9.0mx9.0m column grid– 6x3 spans – C30/37

5.6 Conclusions

In this section, the conclusions obtained from the study are summarised, followed by recommendations for designers to design flat slabs with low embodied carbon content. It is concluded by a discussion about how this study informs a potential review of current design practice.

5.6.1 Conclusions

The study presented in this chapter has investigated the impact of two design choices relating to the design of flat slabs on their embodied carbon content: the choice of concrete grade and the selection of a slab depth.

Regarding the choice of the concrete grade, for slabs of equal depth, those using higher grades have been constantly found to be more carbon intensive: using a higher grade of concrete requires a larger amount of minimum reinforcement to be provided which, associated with the greater carbon intensity of the concrete itself, outweighs the potential savings in the quantities of reinforcement required. Although slabs using higher concrete grades may be shallower and thus limit the design load on the supporting structure, from the cases considered, it is not thought that this reduction in load will result in such savings in carbon in the supporting structure that they compensate for the increased carbon content of the slabs themselves.

The choice of the depth of a slab is found to have a potentially significant impact on its embodied carbon content. Importantly, it is demonstrated that, despite the large uncertainties which may exist in the materials carbon intensities at the time when a slab is designed, it is possible to highlight a span, or a limited range of spans, which results in a low embodied carbon design.

As a consequence, guidance to designers could be given in the form of span-to-depth ratios to target as a function of the design situation considered. Parameters to define these target ratios would include the span of the slab, its design loading and its end usage. For the configurations considered in the study, the size of the slab – its number of bays – was found to have a limited impact on the slab depths to target, but could be included in the list of parameters for greater accuracy.

The study was also the opportunity to investigate the effect of detailing the reinforcement in a slab by only using a standardised finite set of bars and a single spacing between bars. The effect is found to be significant: on the range of spans and depths studied, it is found that detailing using an infinite number of bar sizes and spacings would result in a reduction in provided steel quantities of between 10% and 30%, which corresponds to a reduction of between 4% and 11% in the quantities of carbon embodied in the slabs.

5.6.2 Recommendations for designers

For each design situation, there exists a slab depth resulting in a least embodied carbon design. Optimal depths have been found to be closer to shallow than thick slabs: for residential applications, span-to-effective-depth ratios of around 34, 33 and 28 have been found to result in best results for spans of respectively 6.0m, 7.5m and 9.0m.

In a case where the recommended slab depth is so thin that the slab fails in punching shear, other options should be considered before increasing the depth of the slab. In particular, the introduction of column heads, or an increase in the size of the supporting columns, should be considered: in view of the relative proportions of embodied carbon in the slabs and in the columns highlighted in Figure 40, increasing the size of the columns is preferable to an increase in the slab depth.

Additionally, as far as embodied carbon is concerned, the depth of a slab should not be increased from the recommended optimal value to limit the number of perimeters of punching shear links required above the columns.

Choosing the depth of a slab on the basis of the worst case scenario should be avoided: an example of this was shown by an actual slab which, due to the plan arrangement of the building, had a few of its spans simply-supported. Choosing the depth of the whole slab for this worst-case condition instead of introducing a step in the slab was found to result in an increase in embodied carbon of as much as 13%. Opportunities should be sought to bring the slab as close to its optimal depth as possible for each of the configurations it displays.

5.6.3 Informing a review of current practice

The study has made obvious a series of characteristics to be sought for in the design of flat slabs in order to minimise their embodied carbon content. It highlights a number of aspects which are worth challenging in a design to reduce its embodied carbon content.

Such studies may be developed for a number of structural systems, and could serve as a guiding framework for a review of contemporary buildings aimed at assessing the potential savings achievable if these were designed and built with the primary aim of minimising embodied carbon. This may result in highlighting the need for introducing regulations covering the embodied environmental impact of buildings.

A review of contemporary buildings would also point out typical design behaviours, the effect on embodied carbon of which could be assessed and used to exemplify the impact of certain design decisions among designers.

Chapter 6
Conclusion and further work

This chapter concludes the present thesis. It commences with a review of the contributions made in the work presented. Recommendations are then put forward for future research.

6.1 KEY CONTRIBUTIONS

This thesis has been the opportunity to explore the broad ranging topic of the sustainable use of structural materials to reduce the initial embodied environmental impact of building structures.

This started, in Chapter 1, by considering the wider context of reducing the overall environmental impact associated with the built environment. A review of current environmental issues, and of the contribution of the built environment to those, demonstrated the importance of addressing the topic of the embodied environmental impact of buildings. Options to reduce this impact were reviewed. This accounted for aspects ranging from *life time structural engineering* – what happens to buildings after they are built – to issues related to *eco-efficiency*. Narrowing the focus on the sustainable use of structural materials to reduce the embodied environmental impact of building structures, a framework for investigation into this topic was proposed which highlighted three main areas of investigation:

- Developing an environmentally conscious design practice,
- Refined design criteria and design standards, and
- Technological change.

Two case studies relating to respectively the third and first points of the above framework were presented.

In Chapters 2 and 3, the first study was presented which investigates the shape optimisation of steel I-beams. Motivated by the work of a rolling mill manufacturer who is currently developing a method to roll steel I-beams with a section which varies along the length of the beam, it provided a first understanding of the geometries of the beams which such a rolling mill should target, discussing the savings in steel weight to be obtained from an increasing complexity in the beam geometry with the corresponding complexity in the required rolling mill.

The study found that using bespoke steel I-beams with varying section has the potential to yield significant material savings over currently standard constant catalogue section beams proposed by steel manufacturers. Focusing on a particular type of section, which are able to develop their full elastic capacity but unable to reach their plastic moment resistance, for a design configuration largely representative of contemporary office building design, a weight reduction of at least 33% was shown possible. This result exemplifies the potential merits of using more material-efficient structural element shapes in reducing the embodied environmental impact of building structures.

The study pointed out the disadvantage associated with standardisation as a significant proportion of the total potential savings obtainable by varying the section of a beam were shown to result from the use of a bespoke constant section beam the dimensions of which do not vary along the length of the beam, but are optimally chosen for the design situation considered.

Overall, the results were found to follow the law of diminishing returns as the additional weight savings to be obtained by increasing the number of dimensions varying along a beam's length were shown to reduce as the number of dimensions varying increases. Varying more than two dimensions concurrently was in fact found to produce a marginal improvement, unlikely to justify the corresponding increase in the complexity of the rolling mill. A rolling mill producing beams with a varying flange thickness would yield the majority of the achievable spacings for the two building typologies which, in the UK, consume the largest proportion of steel sections.

Chapters 4 and 5 presented the second case study, dedicated to an investigation into the potential for reducing, through design, the quantities of carbon embodied in a widely used flooring system: flat slabs. In particular, the study looked into the influence of two design choices: the slab depth and the grade of concrete used. Additionally, the impact of using a finite set of bar sizes at a spacing kept constant throughout the slab, on the quantities of reinforcement provided, and on the quantities of embodied carbon, was discussed.

Regarding the choice of the concrete grade, for slabs of equal depth, those using higher grades were constantly found to be more carbon intensive. And although slabs using higher concrete grades may be shallower, and thus limit the design loads seen by the supporting structure, this reduction was shown to be unlikely to result in such savings in carbon in the supporting structure that they compensate for the increased carbon content of the slabs themselves.

The choice of the depth of a slab was found to have a potentially significant impact on its embodied carbon content. Importantly, it was demonstrated that, despite the large uncertainties which may exist in the materials carbon intensities at the time when a slab is designed, it is possible to highlight a span, or a limited range of spans, which results in a low embodied carbon design.
As a consequence, guidance to designers could be given in the form of span-to-depth ratios to target as a function of the design situation considered. Parameters to define these target ratios would include the span of the slab, its design loading and its end usage. For the configurations considered in the study, the size of the slab – its number of bays – was found to have a limited impact on the slab depths to target, but could be included in the list of parameters for greater accuracy.

The study resulted in a first set of recommendations for designers regarding ways to design flat slabs to minimise their embodied carbon content.

Concerning the effect of detailing the reinforcement in a slab by only using a standardised finite set of bars and a single spacing between bars, it was found to be significant: on the range of spans and depths studied, detailing using an infinite number of bar sizes and spacings was shown to result in a reduction in provided steel quantities of between 10% and 30%, corresponding to a reduction of between 4% and 11% in the quantities of carbon embodied in the slabs. Although this is a theoretical result, it highlights the potential merits in limiting the rationalisation of the reinforcement provided.

6.2 RECOMMENDATIONS FOR FURTHER WORK

This section focuses on possible future work on the topic of the *sustainable use of structural materials to reduce the initial embodied environmental impact of building structures*.

Regarding the work presented on the shape optimisation of steel I-beams, a number of possible areas for future work have been described in detail in the corresponding chapter. These include further studies to fully assess the potential merits of using customised constant section beams through the testing of a range of actual design situations, accounting for other section classes than the subset considered in the present study, and adequately accounting for shear web buckling to obtain definite results regarding beams in which the web dimensions vary along the length.

The potential for reductions in environmental impact through other aspects than pure structural weight, in particular through greater integration between structure and services, would constitute an interesting study. The development of an adequate manufacturing process constitutes an obvious area of research in light of the magnitude of the weight savings highlighted. Finally, the issue of the real-life deployment of such a technology constitutes an equally important challenge which needs to be addressed.

This study was motivated by the realisation that the growing relative importance of the embodied environmental impact of buildings in their whole life cycle, and the increasing importance given to environmental issues in general, constitute a new driver for developing more material-efficient structural systems, despite the likely corresponding increase fabrication and construction complexities. Several studies point out the benefits to be obtained, including that presented here, and the development of similar manufacturing and construction technologies for other types of structural element types should be investigated.

In light of the large contribution of the flooring system in the overall embodied carbon in a structure, a method to build more material-efficient reinforced concrete slabs would yield significant benefits. Dombernowsky and Søndergaard (2009) started investigating this topic by developing and building a topology optimised reinforced concrete plate. The shape adopted is extremely complex, requiring a substantial increase in construction complexity. The study of steel I-beams presented here highlights that, from a certain point, further refining the shape of the element only yields limited savings. A similar study into the construction of slabs, balancing obtainable savings with construction complexity, would have much value.

The study of the potential for reducing the quantities of embodied carbon in flat slabs through design resulted in a first set of recommendations for designers and more generally highlighted that significant benefits may be obtained from making the most of currently available design, fabrication and construction methods. Producing guidance for designers to help them making informed choices to minimise the embodied environmental impact of their work would enable the development of a more environmentally-conscious practice. Applying the results obtained to a review of contemporary buildings would enable the assessment of the savings achievable immediately, and would highlight typical design behaviours, the effect on the environmental impact of buildings of which could be assessed and used to exemplify the impact of certain design decisions among designers.

The two studies presented in this thesis have focused on only two of the areas of investigation highlighted in the framework proposed for investigation. *Refining design criteria and design standards* is an obvious way towards less environmentally damaging structures. This is a very broad ranging topic which, as previously discussed, presents many opportunities for improvement.

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APPENDICES

Appendix A
Initial topic

Engineering Doctorate in Building Structures for the 21st Century – Putting the material in the right place

Overview

At any given moment in history the most appropriate building technology will depend fundamentally on the availability and "cost" of: suitable human resources, material, energy, knowledge, and means of preventing harm. These things change through time and building technology changes as a result. So we see the building technology of the middle ages as quite different to that of the Victorian era as a result of major shifts in the availability of materials available and developments in engineering knowledge. Similarly in the era immediately after the second world war where materials were relatively rare there was a brief period where labour intensive but material-efficient concrete shell structures were viable. In recent years the relatively low costs of materials and preference for simple production and fabrication methods have resulted in building forms that are materially inefficient. We believe that we are moving into a new era where material and energy resources will be more highly valued and also where harm prevention is increasingly important. So great are these shifts that we believe them to require a paradigm shift in building technology greater than any seen since the early part of the industrial revolution. The forms of traditional building elements are typically a result of conventional fabrication and construction practices and not their structural efficiency. It is perhaps not surprising therefore that in typical buildings well over 50% of the structural material works at less than 10% of its capacity. The specific aim of this EngD would be to explore and develop building element forms that would increase the utilisation of the structural material. Appraisal criteria for performance will include material quantity, cost, embodied carbon, thermal-mass, eco-points and the like over the whole life cycle. Aims: To design buildings more efficiently taking full advantage of modern computing capability to:

- Optimise the shape of individual components and structural systems so as to get away from run-of-the-mill inefficient designs,
- Carry out multi-criteria produce optimum designs not only in terms of material-efficiency but also in terms of environmental impact.

Method

The project will variously involve

- A literature research to identify what has and hasn't been done on this subject,
- An industrial consultation to review existing practice, identify early resistance to change,
- Carry out extensive structural modelling and analysis on generic sample buildings,

- Design and build physical prototypes which can then be used a showcase and as proofs of concept to enhance the credibility of the methodology to the outside world.

Outcomes

The outcome *for the student* will be the development of practical research skills and extensive knowledge of a subject area that will serve them well throughout their future career as well as an enhanced business and innovation awareness through the LBS courses.

The outcome *for UCL* would be the development of a core of knowledge and capability in an area of increasing relevance and interest in practice. A collaboration with a cutting-edge industrial partner who is keen to improve the industry currently operates.

The outcome *for Expedition* would be further expertise in multi-criteria optimal design and some physical prototypes to push forward the design & construction of better and more appropriate buildings.

The outcome *for the built environment* would be a new generation of building types and systems appropriate to our era with, we believe, positive consequences for society at large.

Appendix B
Optimal beams dimensions – Unconstrained depth

This appendix presents the dimensions obtained for the optimal beams described in Section 3.2 of the main body of this thesis.

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	5.4																			
T (mm)	8.0																			
D (mm)	561.9																			
B (mm)	187.2																			

Table B 1: Beam section dimensions at 20 segments along length – Customised case – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	5.1																			
T (mm)	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.6	7.4	8.2	8.9	9.5	10.0	10.5	10.9	11.2	11.4	11.6	11.6
D (mm)	523.8																			
B (mm)	150.0																			

Table B 2: Beam section dimensions at 20 segments along length – T varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	6.5																			
T (mm)	6.3																			
D (mm)	137.0	157.6	228.9	291.4	345.5	392.9	434.8	471.9	504.9	534.1	559.9	582.5	602.2	619.2	633.5	645.2	654.5	661.5	666.1	668.4
B (mm)	150.0																			

Table B 3: Beam section dimensions at 20 segment along length – D varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.9																			
T (mm)	8.8																			
D (mm)	508.8																			
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	160.5	170.4	179.1	186.6	192.9	197.8	201.6	204.1	205.3

Table B 4: Beam section dimensions at 20 segments along length – B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	6.5	5.2	3.1	3.0	3.6	4.1	4.5	4.8	5.1	5.3	5.6	5.7	5.9	6.0	6.2	6.2	6.3	6.4	6.4	6.4
T (mm)	6.5																			
D (mm)	137.7	161.2	248.1	318.0	374.7	422.2	462.4	496.8	526.3	551.7	573.7	592.5	608.7	622.4	633.8	643.1	650.5	655.9	659.4	661.2
B (mm)	151.0																			

Table B 5: Beam section dimensions at 20 segments along length – t and D varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	5.5																			
T (mm)	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.7	7.3	7.8	8.3	8.7	9.0	9.3	9.5	9.6	9.7
D (mm)	159.9	201.9	275.5	336.8	390.2	437.6	480.5	519.6	555.5	566.9	567.7	568.9	569.9	570.8	571.6	572.3	572.8	573.2	573.5	573.6
B (mm)	150.0																			

Table B 6: Beam section dimensions at 20 segments along length – T and D varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	5.1																			
T (mm)	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.5	6.9	7.2	7.5	7.7	8.0	8.1	8.3	8.4	8.5	8.5	8.6
D (mm)	523.8																			
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	152.8	161.6	169.2	175.8	181.4	186.2	190.3	193.5	196.1	198.1	199.3	200.0

Table B 7: Beam section dimensions at 20 segments along length – T and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	5.9																			
T (mm)	7.2																			
D (mm)	150.4	179.1	244.4	298.8	346.3	388.7	427.0	462.0	494.1	523.7	551.0	576.2	599.5	613.9	613.9	613.9	613.9	613.9	613.9	613.9
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.7	156.9	161.9	165.7	168.2	169.4

Table B 8: Beam section dimensions at 20 segments along length – D and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.5	2.8	2.7	3.1	3.6	4.1	4.5	4.9	5.0	5.2	5.3	5.4	5.5	5.6	5.7	5.7	5.8	5.9	5.9	5.9
T (mm)	6.4	6.5	6.5	6.5	6.4	6.4	6.4	6.4	6.7	7.0	7.3	7.6	7.8	7.9	8.1	8.2	8.2	8.2	8.2	8.2
D (mm)	191.0	291.1	283.5	320.6	378.3	426.6	467.4	502.2	521.9	537.0	550.4	562.1	572.4	581.5	589.4	596.3	602.1	607.0	611.0	613.8
B (mm)	150.0																			

Table B 9: Beam section dimensions at 20 segments along length – t, T and D varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
T (mm)	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.5	6.9	7.2	7.5	7.7	8.0	8.1	8.3	8.4	8.5	8.5	8.6
D (mm)	523.8																			
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	152.8	161.7	169.3	175.9	181.6	186.4	190.4	193.7	196.3	198.2	199.5	200.1

Table B 10: Beam section dimensions at 20 segments along length – t, T and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	2.8	2.8	2.7	2.9	3.4	3.9	4.3	4.6	4.7	5.0	5.2	5.4	5.6	5.7	5.8	5.9	6.0	6.0	6.1	6.1
T (mm)	7.0																			
D (mm)	299.6	292.2	284.5	305.5	361.5	408.7	448.8	477.9	492.8	517.6	539.4	558.2	574.4	588.1	599.5	608.8	616.2	621.6	625.2	627.0
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	153.4	163.2	163.8	164.0	164.2	164.4	164.5	164.6	164.7	164.8	164.8	164.9	164.9

Table B 11: Beam section dimensions at 20 segments along length – t, D and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	5.5																			
T (mm)	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.7	7.3	7.8	8.3	8.7	9.0	9.3	9.5	9.6	9.7
D (mm)	159.9	201.8	275.5	336.7	390.1	437.8	480.6	519.6	555.3	566.9	567.7	568.9	569.9	570.8	571.6	572.3	572.8	573.2	573.5	573.6
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0

Table B 12: Beam section dimensions at 20 segments along length – T, D and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.4	2.8	2.7	3.1	3.6	4.1	4.5	4.9	5.0	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.8	5.9	5.9	5.9
T (mm)	6.4	6.5	6.5	6.5	6.4	6.4	6.4	6.4	6.5	6.7	6.8	6.9	7.0	7.1	7.1	7.2	7.2	8.2	8.2	8.2
D (mm)	195.1	291.1	283.5	320.6	378.3	426.6	467.4	502.2	522.0	536.9	550.0	561.6	571.8	580.8	588.7	595.6	601.5	607.9	611.9	614.7
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	153.4	157.2	160.4	163.0	165.2	167.0	168.3	169.3	169.8	150.0	150.0	150.0

Table B 13: Beam section dimensions at 20 segments along length – t, T, D and B varying – Depth unconstrained

Appendix C
Optimal beams dimensions –Depth constrained to 454.6mm

This appendix presents the dimensions obtained for the optimal beams described in Section 3.3 of the main body of this thesis in the case where an upper-bound of 454.6mm is imposed on the section depth.

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.3																			
T (mm)	9.7																			
D (mm)	454.6																			
B (mm)	226.0																			

Table C 1: Beam section dimensions at 20 segments along length – Customised case – Depth constrained to 454.6mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.4																			
T (mm)	6.4	6.4	6.4	6.4	6.4	6.4	7.6	8.9	10.2	11.4	12.6	13.7	14.8	15.9	16.8	17.7	18.6	19.4	20.1	20.6
D (mm)	454.6																			
B (mm)	150.0																			

Table C 2: Beam section dimensions at 20 segments along length – T varying – Depth constrained to 454.6mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.3																			
T (mm)	9.8																			
D (mm)	207.1	246.9	338.6	415.9	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6
B (mm)	227.1																			

Table C 3: Beam section dimensions at 20 segments along length – D varying – Depth constrained to 454.6mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.3																			
T (mm)	10.3																			
D (mm)	454.6																			
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	151.3	169.3	186.5	203.0	218.7	233.6	239.0	239.0	239.0	239.0	239.0	239.0

Table C 4: Beam section dimensions at 20 segments along length – B varying – Depth constrained to 454.6mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	2.8	2.8	2.9	3.4	3.9	4.2	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3
T (mm)	9.9																			
D (mm)	305.3	297.9	312.2	364.6	408.7	447.0	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6
B (mm)	227.2																			

Table C 5: Beam section dimensions at 20 segments along length – t and D varying – Depth constrained to 454.6mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.3																			
T (mm)	6.4	6.4	6.4	6.4	6.4	6.8	8.3	9.7	11.0	11.7	12.8	13.9	15.0	16.0	17.0	17.9	18.8	19.6	20.3	20.9
D (mm)	202.0	293.8	401.4	443.9	443.9	444.7	447.6	450.4	453.1	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6
B (mm)	150.0																			

Table C 6: Beam section dimensions at 20 segments along length – T and D varying – Depth constrained to 454.6mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.4																			
T (mm)	6.4	6.4	6.4	6.4	6.4	6.4	6.8	7.4	7.9	8.4	8.8	9.2	9.6	9.9	10.2	10.5	10.8	11.0	11.2	11.4
D (mm)	454.6																			
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	159.6	173.0	185.0	195.9	205.8	214.8	223.0	230.6	237.5	243.8	249.6	254.8	259.5	263.2

Table C 7: Beam section dimensions at 20 segments along length – T and B varying – Depth constrained to 454.6mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.3																			
T (mm)	10.4																			
D (mm)	208.8	257.1	351.9	430.9	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6	454.6
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	150.0	151.8	169.8	187.1	203.5	219.3	234.2	240.8	240.8	240.8	240.8	240.8	240.8

Table C 8: Beam section dimensions at 20 segments along length – D and B varying – Depth constrained to 454.6mm

Appendix D
Optimal beams dimensions –Depth constrained to 412.8mm

This appendix presents the dimensions obtained for the optimal beams described in Section 3.3 of the main body of this thesis in the case where an upper-bound of 412.8mm is imposed on the section depth.

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	3.9																			
T (mm)	11.0																			
D (mm)	412.8																			
B (mm)	255.6																			

Table D 1: Beam section dimensions at 20 segments along length – Customised case – Depth constrained to 412.8mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.0																			
T (mm)	6.5	6.5	6.5	6.5	7.0	8.8	10.5	12.2	13.8	15.3	16.8	18.3	19.6	20.9	22.1	23.2	24.3	25.2	26.1	26.9
D (mm)	412.8																			
B (mm)	151.0																			

Table D 2: Beam section dimensions at 20 segments along length – T varying – Depth constrained to 412.8mm – Shear web buckling not accounted for

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.0																			
T (mm)	6.8	6.8	6.8	6.8	6.8	8.5	10.1	11.8	13.3	14.8	16.2	17.6	18.9	20.1	21.3	22.4	23.1	23.1	23.1	23.1
D (mm)	412.8																			
B (mm)	158.8																			

Table D 3: Beam section dimensions at 20 segments along length – T varying – Depth constrained to 412.8mm – Shear web buckling accounted for

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	3.9																			
T (mm)	11.1																			
D (mm)	231.0	254.5	349.0	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8
B (mm)	256.2																			

Table D 4: Beam section dimensions at 20 segments along length – D varying – Depth constrained to 412.8mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	3.9																			
T (mm)	11.8																			
D (mm)	412.8																			
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	153.6	173.7	193.0	211.5	229.2	246.1	262.1	273.3	273.3	273.3	273.3	273.3	273.3

Table D 5: Beam section dimensions at 20 segments along length – B varying – Depth constrained to 412.8mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	2.8	2.8	2.9	3.4	3.8	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
T (mm)	11.1																			
D (mm)	307.9	300.5	311.1	363.1	406.9	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8
B (mm)	256.1																			

Table D 6: Beam section dimensions at 20 segments along length – t and D varying – Depth constrained to 412.8mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	3.8	3.8																		
T (mm)	6.4	6.4	6.4	6.4	7.7	9.6	11.5	12.4	14.0	15.5	17.0	18.5	19.8	21.1	22.3	23.5	24.5	25.5	26.4	27.1
D (mm)	223.1	331.8	400.9	400.9	403.4	407.3	411.0	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8
B (mm)	150.2																			

Table D 7: Beam section dimensions at 20 segments along length – T and D varying – Depth constrained to 412.8mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.0																			
T (mm)	6.4	6.4	6.4	6.4	6.5	7.3	8.0	8.6	9.2	9.7	10.2	10.6	11.0	11.4	11.7	12.0	12.3	12.5	12.7	12.9
D (mm)	412.8																			
B (mm)	150.0	150.0	150.0	150.0	151.5	169.9	186.1	200.4	213.3	225.0	235.7	245.4	254.4	262.6	270.2	277.1	283.3	289.0	294.2	298.2

Table D 8: Beam section dimensions at 20 segments along length – T and B varying – Depth constrained to 412.8mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	4.0																			
T (mm)	6.4	6.4	6.4	6.4	6.5	7.3	8.0	8.6	9.2	9.7	10.2	10.6	11.0	11.4	11.7	12.0	12.3	12.5	12.7	12.9
D (mm)	412.8																			
B (mm)	150.0	150.0	150.0	150.0	151.5	169.9	186.1	200.4	213.3	225.0	235.7	245.4	254.4	262.6	270.2	277.1	283.3	289.0	294.2	298.2

Table D 9: Beam section dimensions at 20 segments along length – T and B varying – Depth constrained to 412.8mm

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t (mm)	3.9																			
T (mm)	11.9																			
D (mm)	233.4	274.7	375.9	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8	412.8
B (mm)	150.0	150.0	150.0	150.0	150.0	150.0	150.0	153.9	174.0	193.4	211.9	229.6	246.5	262.5	274.1	274.1	274.1	274.1	274.1	274.1

Table D 10: Beam section dimensions at 20 segments along length – D and B varying – Depth constrained to 412.8mm

Appendix E
List of active constraints for beams with more than two dimensions varying concurrently

This appendix lists the constraints which are found to be active in the beams described in Section 3.2 of the main body of this thesis for cases where more than two of the dimensions defining an I-section are allowed to vary along the length of the beam.

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Midspan deflection under quasi-permanent load combination	Active																			
Midspan deflection under imposed loads	-																			
Bending resistance	-	-	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Shear resistance	Active	Active	Active	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Web buckling	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Flange buckling	Active	Active	Active	Active	Active	Active	Active	Active	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table E 1: List of active constraints – t, T and D varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Midspan deflection under quasi-permanent load combination	Active																			
Midspan deflection under imposed loads	-																			
Bending resistance	-	-	-	-	-	-	-	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Shear resistance	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Web buckling	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Flange buckling	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Geometrical constraint 1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table E 2: List of active constraints – t, T and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Midspan deflection under quasi-permanent load combination	Active																			
Midspan deflection under imposed loads	-																			
Bending resistance	-	-	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Shear resistance	Active	Active	Active	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Web buckling	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Flange buckling	-	-	-	-	-	-	-	-	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Geometrical constraint 1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table E 3: List of active constraints – t, D and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Midspan deflection under quasi-permanent load combination	Active																			
Midspan deflection under imposed loads	-																			
Bending resistance	-	-	-	-	-	-	-	-	-	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Shear resistance	Active	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Web buckling	-	-	-	-	-	-	-	-	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Flange buckling	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table E 4: List of active constraints – T, D and B varying – Depth unconstrained

Segment number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Midspan deflection under quasi-permanent load combination	Active																			
Midspan deflection under imposed loads	-																			
Bending resistance	-	-	-	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Shear resistance	Active	Active	Active	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Web buckling	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active
Flange buckling	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	Active	-	-
Geometrical constraint 1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Geometrical constraint 3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table E 5: List of active constraints – t, T D and B varying – Depth unconstrained

Appendix F
Minimum reinforcement area for crack control

The design method implemented for the design of the slabs of the study presented in Chapter 4 is described in Section 4.3 of the main body of this thesis. In the present section, the method used to calculate the minimum reinforcement area to be provided for crack control is further detailed.

For a definition of the variables used, the reader is referred to Figure 39 at the beginning of Chapter 4.

The requirements for the minimum reinforcement area to be provided in the slab for crack control are covered in Clause 7.3.2(2) of EN1992-1-1 as the following:

$$A_{S,crack} = \frac{0.4\, f_{ctm} A_{ct}}{f_{yk}}$$

$k_c = 0.4$ for pure flexural members.

$f_{ct,eff}$ can be taken as $f_{ct,m}$ according to Clause 7.1(2) of EN1992-1-1.

$f_{ct,eff} = 0.30 \times f_{ck}{}^{2/3}$ for concrete grades lower than C50/60 as per Table 3.1 of EN1992-1-1.

A_{ct} is the area of concrete within the tensile zone. The tensile zone is defined as that part of the section which is calculated to be in tension just before the formation of the first crack.

The strain and stress distributions in the section just before the formation of the first crack are shown on Figure F 1.

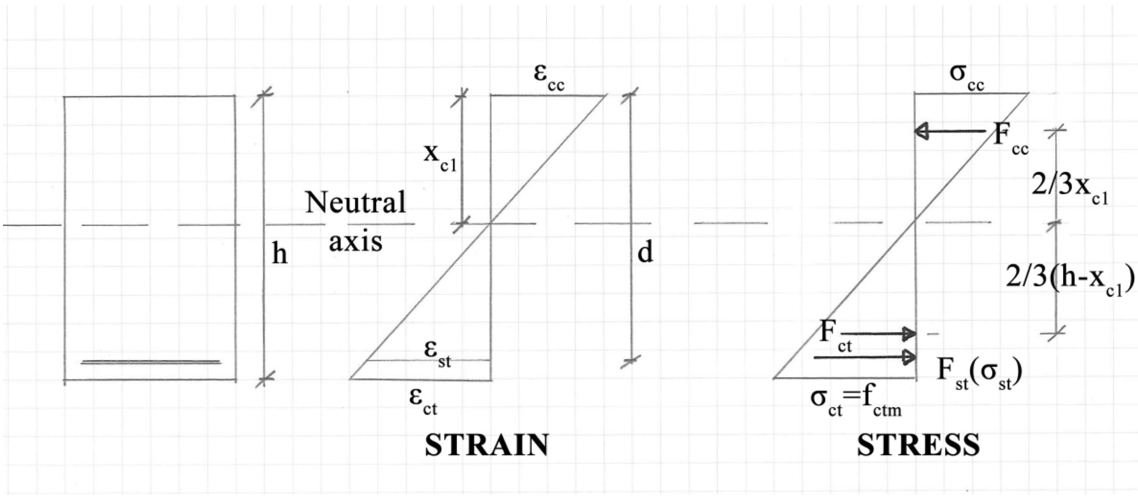

Figure F 1: Strain and stress distribution in section just before the formation of the first crack

$\sigma_{ct} = f_{ctm}$ as the section is on the point of cracking.

The strain distribution in the section is linear as the section behaves elastically. Consequently:

$$\varepsilon_{cc} = \frac{x_{1c}}{h - x_{1c}} \varepsilon_{ct}$$

$$\varepsilon_{st} = \frac{d -_{1c}}{h - x_{1c}} \varepsilon_{ct}$$

Additionally:

$$\varepsilon_{ct} = \frac{\sigma_{ct}}{E_{c,eff,1c}}$$

The total resultant force in the part of the concrete under compression can be expressed as:

$$F_{cc} = \frac{x_{1c} \times 1000 \times \sigma_{cc}}{2}$$
$$F_{cc} = \frac{x_{1c} \times 1000 \times E_{c,eff,1c} \times \varepsilon_{cc}}{2}$$
$$F_{cc} = \frac{{x_{1c}}^2 \times 1000 \times E_{c,eff,1c} \times \varepsilon_{ct}}{2 \times (h - x_{1c})}$$
$$F_{cc} = \frac{{x_{1c}}^2 \times 1000 \times f_{ctm}}{2 \times (h - x_{1c})}$$

The total resultant force in the part of the concrete under tension can be expressed as:

$$F_{ct} = \frac{(h - x_{1c}) \times 1000 \times f_{ctm}}{2}$$

The force in the reinforcement can be expressed as:

$$F_{st} = A_{s,prov} \times \sigma_{st}$$

$$F_{st} = A_{s,prov} \times \varepsilon_{st} \times E_s$$

$$F_{st} = A_{s,prov} \times E_s \times \frac{d - x_{1c}}{h - x_{1c}} \times \varepsilon_{ct}$$
$$F_{st} = \frac{A_s \times E_s \times (d - x_{1c}) \times f_{ctm}}{(h - x_{1c}) \times E_{c,eff,1c}}$$
$$F_{st} = \frac{A_s \times (d - x_{1c}) \times f_{ctm} \times \alpha_{1c}}{(h - x_{1c})}$$

For axial equilibrium to be verified in the section, the following relationship must be verified:

$$F_{cc} = F_{ct} + F_{st}$$

$$\frac{{x_{1c}}^2 \times 1000 \times f_{ctm}}{2 \times (h - x_{1c})} = \frac{(h - x_{1c}) \times 1000 \times f_{ctm}}{2} + \frac{A_{S,prov} \times (d - x_{1c}) \times f_{ctm} \times \alpha_{1c}}{(h - x_{1c})}$$
$$\frac{{x_{1c}}^2 \times 1000}{2} = \frac{(h - x_{1c})^2 \times 1000}{2} + A_{S,prov} \times (d - x_{1c}) \times \alpha_{1c}$$
$$500 \times (h^2 - 2 \times h \times x_{1c}) + A_{S,prov} \times d \times \alpha_{1c} - A_{S,prov} \times x_{1c} \times \alpha_{1c} = 0$$

$$x_{1c}\left(1000 \times h + A_{S,prov} \times \alpha_{1c}\right) = 500 \times h^2 + A_{S,prov} \times d \times \alpha_{1c}$$

$$x_{1c} = \frac{500 \times h^2 + A_{S,prov} \times d \times \alpha_{1c}}{1000 \times h + A_{S,prov} \times \alpha_{1c}}$$

Hence:

$$A_{ct} = 1000 \times (h - x_{1c})$$

$$A_{ct} = 1000 \times \left(h - \frac{500 \times h^2 + A_{S,prov} \times d \times \alpha_{1c}}{1000 \times h + A_{S,prov} \times \alpha_{1c}}\right)$$

Finally:

$$A_{S,crack} = \frac{0.4 \times f_{ctm} \times 1000 \times \left(h - \frac{500 \times h^2 + A_{S,prov} \times d \times \alpha_{1c}}{1000 \times h + A_{S,prov} \times \alpha_{1c}}\right)}{f_{yk}}$$

The choice of α_{1c} is not obvious as it is not known when the section will form its first crack and hence what the modulus of elasticity of concrete will be.

As α decreases, $A_{S,crack}$ increases. As a consequence, using a low value for α_{1c} is conservative. The value of the secant modulus of C40/50 concrete, which is the highest concrete grade considered in this investigation is used: $E_{cm} = 35GPA$. α_{1c} is hence taken as 5.7 which is conservative. It is worth noting that in all of the cases tested, the requirements for minimum reinforcement from clause 9.2.1.1(1) of EN1992-1-1 (BSI 2004b) were found to be more onerous than the requirements for minimum reinforcement for crack control described here.

Appendix G
Calculation of lever arm of cracked section under elastic stress distribution

The design method implemented for the design of the slabs of the study presented in Chapter 4 is described in Section 4.3 of the main body of this thesis. In the present section, the method used to calculate the lever arm of a cracked section under elastic stress distribution is further detailed, as this has an impact on the maximum allowed spacing between bars and on the limiting span-to-effective depth ratio..

For a definition of the variables used, the reader is referred to Figure 39 at the beginning of Chapter 4 and in Appendix F.

The maximum allowable bar spacing to ensure crack widths are kept below the limit of 0.4mm specified in Table NA.4 of the National Annex to EN1992-1-1 (BSI 2004c) is defined in Table 7.3N of EN1992-1-1 as:

$$S_{max} = 500 - \frac{\sigma_{S,SLS}}{0.8}$$

As detailed in Figure 42, the limiting span-to-effective-depth ratio can be calculated as:

$$\frac{l}{d_{limit}} = \frac{310}{\sigma_{S,SLS}} \times \frac{l}{d_{limit,span}}$$

$\sigma_{S,SLS}$, which appears in both calculations, can be calculated as:

$$\sigma_{S,SLS} = \frac{M_{SLS}}{z_{SLS} \times A_{S,prov}}$$

To work out z_{SLS}, the section is assumed to be cracked as this results in a greater stress in the reinforcement and consequently in a reduced allowable spacing between bars. The strain and stress distributions in the section are given on Figure G 1.

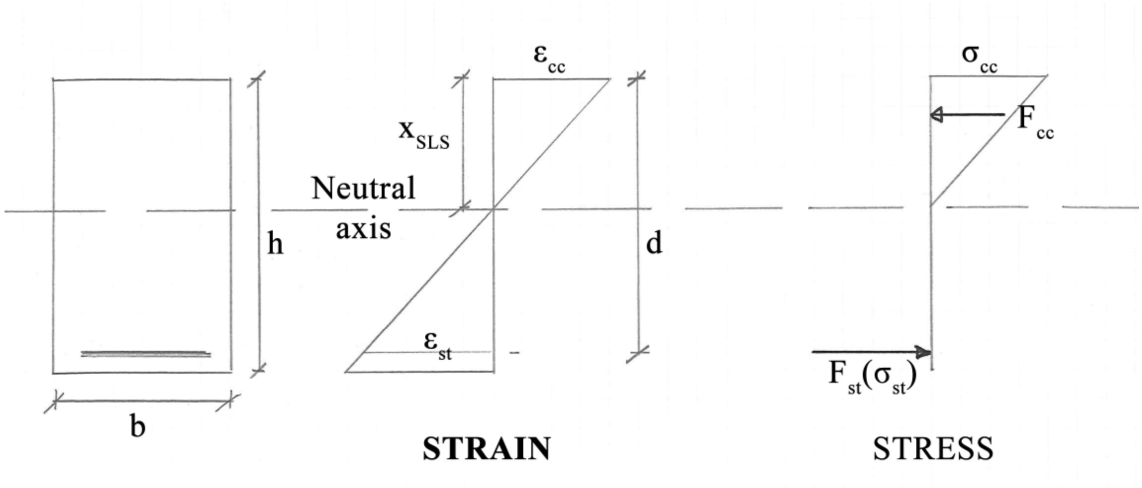

Figure G 1: Strain and stress distribution in cracked section

The tensile force in the reinforcement and the resultant compressive force of compressive stresses in concrete can be expressed as follows:

$$F_{st} = A_{s,prov} \times \varepsilon_{st} \times E_s$$

$$F_{cc} = \frac{E_{c,eff} \times \varepsilon_{cc} \times x_{SLS} \times b}{2}$$

The strain distribution in the section is linear as the section behaves elastically. Consequently:

$$\frac{\varepsilon_s}{d - x_{SLS}} = \frac{\varepsilon_{cc}}{x_S}$$

Hence:

$$F_{st} = E_s \frac{d - x_{SLS}}{x_{SLS}} \times \varepsilon_{cc} \times A_{s,prov}$$

Writing the axial equilibrium of forces in the section results in the following equation:

$$F_{cc} = F_{st}$$

$$\frac{E_{c,eff} \times \varepsilon_{cc} \times x_{SLS} \times b}{2} = E_s \frac{d - x_{SLS}}{x_{SLS}} \times \varepsilon_{cc} \times A_{s,prov}$$

$$\frac{b}{2} \times {x_{SLS}}^2 + \alpha \times A_{S,prov} \times x_{SLS} - \alpha \times A_{S,prov} \times d = 0$$

Solving this quadratic equation results in the following depth of neutral axis:

$$x_{SLS} = \frac{-2 \times \alpha \times A_{S,prov} + \sqrt{4 \times \alpha^2 \times {A_{S,prov}}^2 + 8 \times b \times \alpha \times A_{S,prov} \times d}}{2 \times b}$$

And consequently:

$$z_{SLS} = d - \frac{x}{3}$$

$$z_{SLS} = d + \frac{2 \times \alpha \times A_{S,prov} - \sqrt{4 \times \alpha^2 \times {A_{S,prov}}^2 + 8 \times b \times \alpha \times A_{S,prov} \times d}}{6 \times b}$$

A value needs to be assigned to the modular ratio. Here, as α increases, z_{SLS} reduces and $\sigma_{S,SLS}$ increases. As a consequence, both the maximum allowable spacing between bars and the limiting span-to-effective depth ratio reduce. Consequently, using a large value for α is conservative.

From Figure 3.1 of EN1992-1-1, it appears unlikely that

$$E_{c,eff} < \frac{E_{cm}}{3}$$

α is hence calculated as

$$\frac{3 \times E_s}{E_{cm}}$$

Appendix H
Reinforcement provided in slabs

The following tables give the reinforcement provided in each slab for each of the spans and concrete grades considered in the study presented in Chapter 5.

Slab depth	Basic mesh	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
(mm)	(mm)	Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
300	H12@200	H12@200	H10@200	-	-	H10@200	-
275	H10@200	H16@200	H10@200	H10@200	-	H10@200	H10@200
250	H10@200	H16@200	H12@200	H10@200	-	H10@200	H10@200
225	H10@200	H16@200	H12@200	H10@200	-	H10@200	H10@200
205	H10@200	H20@200	H16@200	H10@200	-	H12@200	H10@200

Table H 1: Reinforcement provided in slab supported on a 6.0mx6.0m column grid and using concrete grade C30/37

Slab depth	Basic mesh	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
(mm)	(mm)	Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
375	H12@200	H16@200	H12@200	-	-	H10@200	H10@200
350	H12@200	H16@200	H12@200	-	-	H10@200	H10@200
325	H12@200	H16@200	H12@200	H10@200	-	H10@200	H10@200
300	H12@200	H20@200	H16@200	H10@200	-	H10@200	H10@200
275	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
265	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
260	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200

Table H 2: Reinforcement provided in slab supported on a 7.5mx7.5m column grid and using concrete grade C30/37

Slab depth	Basic mesh	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
(mm)	(mm)	Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
450	H16@200	H16@200	H10@200	-	-	H10@200	-
425	H16@200	H16@200	H10@200	-	-	H10@200	-
400	H12@200	H20@200	H16@200	H10@200	-	H12@200	H10@200
375	H12@200	H20@200	H16@200	H10@200	H10@200	H12@200	H10@200
350	H12@200	H25@200	H20@200	H10@200	H10@200	H16@200	H10@200
325	H12@200	H25@200	H20@200	H10@200	H10@200	H20@200	H10@200

Table H 3: Reinforcement provided in slab supported on a 9.0mx9.0m column grid and using concrete grade C30/37

Slab depth	Basic mesh	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
(mm)	(mm)	Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
375	H12@200	H16@200	H12@200	-	-	H10@200	H10@200
350	H12@200	H16@200	H12@200	-	-	H10@200	H10@200
325	H12@200	H16@200	H12@200	H10@200	-	H10@200	H10@200
300	H10@200	H20@200	H16@200	H10@200	H10@200	H12@200	H10@200
275	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
265	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
260	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200

Table H 4: Reinforcement provided in slab supported on a 7.5mx7.5m column grid and using concrete grade C28/35

Slab depth	Basic mesh	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
(mm)	(mm)	Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
375	H16@200	H10@200	H10@200	-	-	-	-
350	H12@200	H16@200	H12@200	-	-	H10@200	H10@200
325	H12@200	H16@200	H12@200	H10@200	-	H10@200	H10@200
300	H12@200	H16@200	H16@200	H10@200	-	H10@200	H10@200
275	H12@200	H20@200	H16@200	H10@200	-	H12@200	H10@200
260	H10@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200
255	H10@200	H20@200	H16@200	H10@200	H10@200	H20@200	H10@200

Table H 5: Reinforcement provided in slab supported on a 7.5mx7.5m column grid and using concrete grade C35/45

Slab depth	Basic mesh	Hogging reinforcement				Sagging reinforcement	
		Column strip		Middle strip		Column and middle strips	
(mm)	(mm)	Above first interior support	Above interior supports	Above first interior support	Above interior supports	In end span	In inner span
375	H16@200	H10@200	H10@200	-	-	-	-
350	H16@200	H12@200	H10@200	-	-	-	-
325	H12@200	H16@200	H12@200	H10@200	-	H10@200	H10@200
300	H12@200	H16@200	H16@200	H10@200	-	H10@200	H10@200
275	H12@200	H20@200	H16@200	H10@200	H10@200	H12@200	H10@200
265	H12@200	H20@200	H16@200	H10@200	H10@200	H12@200	H10@200
250	H12@200	H20@200	H16@200	H10@200	H10@200	H16@200	H10@200

Table H 6: Reinforcement provided in slab supported on a 7.5mx7.5m column grid and using concrete grade C40/50

Appendix I: Punching shear checks

The following figures show extracts of the spreadsheet used to check punching shear and design the shear reinforcement to be provided in the slabs considered for investigation in Section 0.

PUNCHING SHEAR REINFORCEMENT FOR INNER COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

Slab					**Column**		
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)		h (mm)	b (mm)	
151	167	159.0	30		350	350	
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x				
H10	H16	200	1500	mm	**Loading**		
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y		Ved (kN)	β	Veff (kN)
H10	H16	200	1500	mm	475.88	1.15	548

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1400	2.47	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
3398	1.02	0.71

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<79mm → 75mm
Spacing between perimeters<119mm → 100mm
Distance from face of col. to last perimeter>312mm → 4 perimeters required

Asw,req (mm2)	Links used	Number of links required per perimeter
382	H8	8

Perimeter1	Spacing of legs< 233mm →	Chosen spacing:	200
Perimeter2	Spacing of legs< 238mm →	Chosen spacing:	200
Perimeter3	Spacing of legs< 238mm →	Chosen spacing:	200
Perimeter4	Spacing of legs< 318mm →	Chosen spacing:	300

Figure I 1: Punching shear check of 205mm thick slab supported on 6.0mx6.0m column grid at inner column

PUNCHING SHEAR REINFORCEMENT FOR EDGE COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

NOTE: Orientation of column set to 1 if its longest dimension is parallel to the edge. Otherwise set to 0.

Slab				**Column**			
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	h (mm)	b (mm)	dedge(mm)	orientation
151	167	159.0	30	350	350	0	1
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x				
H10	H16	200	1500 mm	**Loading**			
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	Ved (kN)	β	Veff (kN)	
H10	H16	200	1500 mm	237.94	1.25	298	

Special edge reinforcement required, cf. EN 1992-1-1, cl.6.4.2(5)

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0* (mm)	U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1400	827	2.27	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
2049	0.92	0.71

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<79mm → 75mm
Spacing between perimeters<119mm → 100mm
Distance from face of col. to last perimeter>268mm → 3 perimeters required

Links used
H8

Perimeter1	Configuration1	Spacing of legs< 238mm →	Chosen spacing:	200mm
Perimeter2	Configuration1	Spacing of legs< 238mm →	Chosen spacing:	200mm
Perimeter3	Configuration1	Spacing of legs< 238mm →	Chosen spacing:	200mm

Figure I 2: Punching shear check of 205mm thick slab supported on 6.0mx6.0m column grid at edge column

PUNCHING SHEAR REINFORCEMENT FOR CORNER COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

Slab				Column		Dist. from edge	
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	c1 (mm)	c2 (mm)	d1 (mm)	d2 (mm)
151	167	159.0	30	350	350	0	0
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x				
H10	H16	200	1500 mm	Loading			
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	Ved (kN)	β	Veff (kN)	
H10	H16	200	1500 mm	118.97	1.5	179	

Special edge reinforcement required, cf. EN 1992-1-1, cl.6.4.2(5)

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0* (mm)	U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1400	477	2.37	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
1200	0.94	0.71

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<79mm → 75mm

Spacing between perimeters<119mm → 100mm

Distance from face of col. to last perimeter>326mm → 4 perimeters required

Links used

H8

Perimeter1	Configuration1	Spacing of legs< 238mm →	Chosen spacing: 200mm
Perimeter2	Configuration1	Spacing of legs< 238mm →	Chosen spacing: 200mm
Perimeter3	Configuration1	Spacing of legs< 238mm →	Chosen spacing: 200mm
Perimeter4	Configuration1	Spacing of legs< 318mm →	Chosen spacing: 300mm

Figure I 3: Punching shear check of 205mm thick slab supported on 6.0mx6.0m column grid at corner column

PUNCHING SHEAR REINFORCEMENT FOR INNER COLUMNS

expedition

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

DATA

Slab				Column		
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	h (mm)	b (mm)	
206	222	214.0	30	400	400	
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x	**Loading**		
H10	H16	200	1875 mm	Ved (kN)	β	Veff (kN)
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	847.97	1.15	976
H10	H16	200	1875 mm			

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1600	2.86	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
4289	1.07	0.63

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<107mm → 100mm
Spacing between perimeters<161mm → 100mm
Distance from face of col. to last perimeter>577mm → 6 perimeters required

Asw,req (mm2)	Links used	Number of links required per perimeter
563	H8	12

Perimeter1	Spacing of legs< 185mm →	Chosen spacing:	150
Perimeter2	Spacing of legs< 238mm →	Chosen spacing:	200
Perimeter3	Spacing of legs< 290mm →	Chosen spacing:	250
Perimeter4	Spacing of legs< 321mm →	Chosen spacing:	300
Perimeter5	Spacing of legs< 395mm →	Chosen spacing:	350
Perimeter6	Spacing of legs< 428mm →	Chosen spacing:	400

Figure I 4: Punching shear check of 260mm thick slab supported on 7.5mx7.5m column grid at inner column

PUNCHING SHEAR REINFORCEMENT FOR EDGE COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

NOTE: Orientation of column set to 1 if its longest dimension is parallel to the edge. Otherwise set to 0.

Slab					**Column**			
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)		h (mm)	b (mm)	dedge(mm)	orientation
206	222	214.0	30		400	400	0	1
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x					
H10	H16	200	1875 mm		**Loading**			
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y		Ved (kN)	β	Veff (kN)	
H10	H16	200	1875 mm		423.98	1.25	530	

Special edge reinforcement required, cf. EN 1992-1-1, cl.6.4.2(5)

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0* (mm)	U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1600	1042	2.38	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
2545	0.98	0.63

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<107mm → 100mm
Spacing between perimeters<161mm → 100mm
Distance from face of col. to last perimeter>549mm → 6 perimeters required

Links used
H8

Perimeter1	Configuration1	Spacing of legs< 302mm →	Chosen spacing: 300mm
Perimeter2	Configuration1	Spacing of legs< 321mm →	Chosen spacing: 300mm
Perimeter3	Configuration1	Spacing of legs< 321mm →	Chosen spacing: 300mm
Perimeter4	Configuration1	Spacing of legs< 321mm →	Chosen spacing: 300mm
Perimeter5	Configuration1	Spacing of legs< 428mm →	Chosen spacing: 400mm
Perimeter6	Configuration1	Spacing of legs< 428mm →	Chosen spacing: 400mm

Figure I 5: Punching shear check of 260mm thick slab supported on 7.5mx7.5m column grid at edge column

PUNCHING SHEAR REINFORCEMENT FOR CORNER COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

Slab				Column		Dist. from edge	
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	c1 (mm)	c2 (mm)	d1 (mm)	d2 (mm)
206	222	214.0	30	400	400	0	0
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x				
H10	H16	200	1875 mm	Loading			
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	Ved (kN)	β	Veff (kN)	
H10	H16	200	1875 mm	211.99	1.5	318	

Special edge reinforcement required, cf. EN 1992-1-1, cl.6.4.2(5)

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0* (mm)	U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1600	642	2.32	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
1472	1.01	0.63

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<107mm → 100mm
Spacing between perimeters<161mm → 100mm
Distance from face of col. to last perimeter>672mm → 7 perimeters required

Links used
H8

Perimeter1	Configuration1	Spacing of legs< 319mm	→ Chosen spacing:	300mm
Perimeter2	Configuration1	Spacing of legs< 321mm	→ Chosen spacing:	300mm
Perimeter3	Configuration1	Spacing of legs< 321mm	→ Chosen spacing:	300mm
Perimeter4	Configuration1	Spacing of legs< 321mm	→ Chosen spacing:	300mm
Perimeter5	Configuration1	Spacing of legs< 428mm	→ Chosen spacing:	400mm
Perimeter6	Configuration1	Spacing of legs< 428mm	→ Chosen spacing:	400mm
Perimeter7	Configuration1	Spacing of legs< 428mm	→ Chosen spacing:	400mm

Figure I 6: Punching shear check of 260mm thick slab supported on 7.5mx7.5m column grid at corner column

PUNCHING SHEAR REINFORCEMENT FOR INNER COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

Slab

dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	
260	280	270.0	30	
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x	
H12	H20	200	2250	mm
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	
H12	H20	200	2250	mm

Column

h (mm)	b (mm)
450	450

Loading

Ved (kN)	β	Veff (kN)
1398.8	1.15	1609

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1800	3.32	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
5193	1.15	0.64

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<135mm → 100mm

Spacing between perimeters<203mm → 200mm

Distance from face of col. to last perimeter>791mm → 5 perimeters required

Asw,req (mm2)	Links used	Number of links required per perimeter
1462	H8	30

Perimeter1	Spacing of legs< 80mm	→ Chosen spacing:	75
Perimeter2	Spacing of legs< 122mm	→ Chosen spacing:	100
Perimeter3	Spacing of legs< 164mm	→ Chosen spacing:	150
Perimeter4	Spacing of legs< 206mm	→ Chosen spacing:	200
Perimeter5	Spacing of legs< 248mm	→ Chosen spacing:	200

Figure I 7: Punching shear check of 325mm thick slab supported on 9.0mx9.0m column grid at inner column

PUNCHING SHEAR REINFORCEMENT FOR EDGE COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

NOTE: Orientation of column set to 1 if its longest dimension is parallel to the edge. Otherwise set to 0.

Slab				Column			
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	h (mm)	b (mm)	dedge(mm)	orientation
260	280	270.0	30	450	450	0	1
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x				
H12	H20	200	2250 mm	**Loading**			
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	Ved (kN)	β	Veff (kN)	
H12	H20	200	2250 mm	699.38	1.25	875	

Special edge reinforcement required, cf. EN 1992-1-1, cl.6.4.2(5)

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0* (mm)	U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1800	1260	2.58	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
3046	1.07	0.64

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<135mm	→	100mm
Spacing between perimeters<203mm	→	200mm
Distance from face of col. to last perimeter>778mm	→	5 perimeters required

Links used
H8

Perimeter1	Configuration1	Spacing of legs< 110mm	→	Chosen spacing:	100mm
Perimeter2	Configuration1	Spacing of legs< 152mm	→	Chosen spacing:	150mm
Perimeter3	Configuration1	Spacing of legs< 194mm	→	Chosen spacing:	200mm
Perimeter4	Configuration1	Spacing of legs< 236mm	→	Chosen spacing:	250mm
Perimeter5	Configuration1	Spacing of legs< 278mm	→	Chosen spacing:	250mm

Figure I 8: Punching shear check of 325mm thick slab supported on 9.0mx9.0m column grid at edge column

PUNCHING SHEAR REINFORCEMENT FOR CORNER COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	26/07/2012		

expedition

DATA

Slab				Column		Dist. from edge	
dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	c1 (mm)	c2 (mm)	d1 (mm)	d2 (mm)
260	280	270.0	30	450	450	0	0
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x				
H12	H20	200	2250 mm	Loading			
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	Ved (kN)	β	Veff (kN)	
H12	H20	200	2250 mm	349.69	1.5	525	

Special edge reinforcement required, cf. EN 1992-1-1, cl.6.4.2(5)

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0* (mm)	U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1800	810	2.41	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
1748	1.12	0.64

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<135mm → 100mm
Spacing between perimeters<203mm → 200mm
Distance from face of col. to last perimeter>957mm → 6 perimeters required

Links used
H8

Perimeter1	Configuration1	Spacing of legs< 117mm →	Chosen spacing:	100mm
Perimeter2	Configuration1	Spacing of legs< 152mm →	Chosen spacing:	150mm
Perimeter3	Configuration1	Spacing of legs< 187mm →	Chosen spacing:	150mm
Perimeter4	Configuration1	Spacing of legs< 222mm →	Chosen spacing:	200mm
Perimeter5	Configuration1	Spacing of legs< 257mm →	Chosen spacing:	250mm
Perimeter6	Configuration1	Spacing of legs< 291mm →	Chosen spacing:	250mm

Figure I 9: Punching shear check of 325mm thick slab supported on 9.0mx9.0m column grid at corner column

PUNCHING SHEAR REINFORCEMENT FOR INNER COLUMNS

JOB NO.		SLAB		PAGE	
JOB		COLUMN		CHD	
BY		DATE	27/07/2012		

expedition

DATA

Slab

dx (mm)	dy (mm)	deff (mm)	fck (N/mm2)	
276	292	284.0	30	
Top1.x	Top2.x	Spacing (mm)	1/2 Col. Strip width x	
H10	H16	200	1875	mm
Top1.y	Top2.y	Spacing (mm)	1/2 Col. Strip width y	
H10	H16	200	1875	mm

Column

h (mm)	b (mm)
400	400

Loading

Ved (kN)	β	Veff (kN)
980.86	1.15	1128

CHECK CONCRETE STRENGTH ADEQUACY - EN1992-1-1, cl.6.4.3(2a)

U0 (mm)	ved,0 (N/mm2)	vrd,c,0 (N/mm2)
1600	2.49	5.28

Resistance of concrete at column perimeter OK

CHECK PUNCHING SHEAR - EN1992-1-1, cl.6.4.3(2b)

U1 (mm)	ved,1 (N/mm2)	vrd,c,1 (N/mm2)
5169	0.77	0.54

Shear reinforcement required

PROVIDING SHEAR REINFORCEMENT - EN1992-1-1, cl.6.4.5

Distance from face of column to 1rst perimeter<142mm	→	100mm
Spacing between perimeters<213mm	→	200mm
Distance from face of col. to last perimeter>490mm	→	3 perimeters required

Asw,req (mm2)	Links used	Number of links required per perimeter
784	H8	16

Perimeter1	Spacing of legs< 139mm	→	Chosen spacing:	100
Perimeter2	Spacing of legs< 217mm	→	Chosen spacing:	200
Perimeter3	Spacing of legs< 296mm	→	Chosen spacing:	250

Figure I 10: Punching shear check on inner column for slab supported on 7.5mx7.5m column grid to ensure number of shear link perimeter is kept at three