Resources, Conservation and Recycling 57 (2011) 48–60

Contents lists available at SciVerse ScienceDirect

Resources, Conservation and Recycling

journal homepage: www.elsevier.com/locate/resconrec

The technical potential for reducing metal requirements through lightweight product design

Mark A. Carruth, Julian M. Allwood *, Muiris C. Moynihan

Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

ARTICLE INFO

Article history:
Received 30 May 2011
Received in revised form 8 September 2011
Accepted 9 September 2011

Keywords:
Lightweight design
Material efficiency
Design principles
CO_2 emissions

ABSTRACT

Metal production consumes around 10% of all global energy, so is a significant driver of climate change and other concerns about sustainability. Demand for metal is rising and forecast to double by 2050 through a combination of growing total demand from developing countries, and ongoing replacement demand in developed economies. Metal production is already extremely efficient, so the major opportunities for emissions abatement in the sector are likely to arise from material efficiency – using less new metal to meet demand for services. Therefore this paper examines the opportunity to reduce requirements for steel and aluminium by lightweight design. A set of general principles for lightweight design are proposed by way of a simple analytical example, and are then applied to five case study products which cumulatively account for 30% of global steel product output. It is shown that exploiting lightweight design opportunities for these five products alone could reduce global steel requirements by 5%, and similar savings in aluminium products could reduce global aluminium requirements by 7%. If similar savings to those in the design case studies were possible in all steel and aluminium products, total material requirements could be reduced by 25–30%. However, many of these light-weighting measures are, at present, economically unattractive, and may take many years to implement.

1. The scale of global metal flows

The global flow of metal is vast. Each year, around 1040 Mt of new steel products and 45 Mt of new aluminium products are produced (Allwood et al., 2011), and per-capita metal stocks in more-developed countries are estimated by Gerst and Graedel (2008) to be around 7 tonnes of steel and 350–500 kg of aluminium. Müller et al. (2011) produce a higher estimate of 8–12 tonnes for steel, noting that in the UK and the US steel stocks have reached a 'plateau' and remain roughly constant. However, even in these countries where total steel stocks are steady, the annual production of new products is 200–400 kg per capita, to replace products which have been discarded or have reached end of life.

Production of both steel and aluminium is energy intensive, and hence a major driver of CO_2 emissions. However, both industries are already extremely efficient – so there is little potential for future reductions in energy inputs per unit of metal output. The IEA (2009) estimate that universal adoption of best practice in industry has the potential to reduce emissions from steelmaking by 13%, and from aluminium by 12%, which is clearly insufficient to meet targeted 50% reductions. One strategy to reduce the overall emissions from these sectors is to use less new material to provide a given service. Therefore this paper asks to what extent the annual production of new material could be reduced by designing lighter weight products?

Because of the diversity of products made from steel and aluminium, it is not possible to analyse the potential for weight saving in all products. Instead, an estimate is made by examining representative case studies. Allwood et al. (2012) tracked the flow of steel and aluminium along their supply chains from liquid metal to end-use, allocated by sector, noting the inputs, outputs and scrap losses at each stage to produce a Sankey diagram of material flow for the two metals. For steel, the construction sector accounted for over 50% of all material use, with vehicles and industrial equipment accounting for a further 25%, and the final 25% attributable to other assorted products; for aluminium, the split between these four sectors is roughly equal. Therefore significant savings in any of these sectors would produce large savings overall.

By looking at specific examples in the large metal consuming sectors described above, this paper has three key aims: firstly, to estimate the potential cut in total requirements for liquid metal that might be achieved by lightweight design; secondly, to devise a set of general principles which could be applied to any product, metallic or otherwise, to maximise material efficiency; thirdly, to identify the technical constraints which may inhibit the application of lightweight designs.

* Corresponding author. Tel.: +44 1223 338181; fax: +44 1223 332662.
E-mail address: jma42@cam.ac.uk (J.M. Allwood).

0921-3449/$ – see front matter
doi:10.1016/j.resconrec.2011.09.018

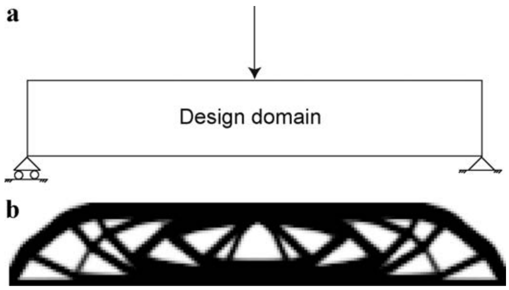


Fig. 1. Structural optimisation example with (a) design domain and (b) example result produced by SIMP.

Although material substitution is an attractive strategy for weight saving in some cases, analysing its effect on CO_2 emissions is difficult, as the whole product lifecycle must be considered. To simplify the analysis, this paper does not consider options for substituting steel or aluminium with alternative materials, such as composites, which is discussed comprehensively by Ashby and Jones (2005). However, options for material upgrade (replacement with higher strength alloys of the same material) are considered.

2. Reducing mass by design

The design of lightweight products has a long history. From formal processes to optimise the design of products for a given service to more general design guides aimed at boosting material efficiency, the literature on designing lightweight products dates back over 100 years.

The field of structural optimisation provides a formal mathematical framework for minimising product weight for a given service, either by using a given mass of material to maximise some performance metric, or by minimising the mass of the product subject to a given performance requirement. The first work in this field was conducted over 100 years ago by Michell (1904), who developed a method for minimising the mass of planar truss structures subjected to a specified loading. (Michell's beautifully elegant structures are mathematically optimal, albeit impractical as they often require an infinite number of elements, so could not in reality be manufactured.) Further work in this field by Rozvany (1972) extended the ideas to 2D grids where the load was applied perpendicular to the plane of the structure. Rossow and Taylor (1973) introduced the use of computerised finite element techniques to structural optimisation, which were used by Bendsøe and Kikuchi (1988) in the development of the "homogenisation" technique, which forms the basis of many modern structural optimisation analyses.

One of the most common approaches to structural optimisation is known as SIMP, introduced by Bendsøe (1989), and is similar to the homogenisation approach of Bendsøe and Kikuchi (1988). In the SIMP approach, a design domain is specified, along with a set of boundary conditions, i.e. support locations and applied loadings, an example of which is shown in Fig. 1a. The domain is decomposed into elements, and the optimisation problem is then established as:

$$\min_{\underline{\boldsymbol{x}}} \; C(\underline{\boldsymbol{x}}) \tag{1}$$

$$\text{subject to} \begin{cases} K(\underline{\boldsymbol{x}})\boldsymbol{u} = \boldsymbol{f}, \\ V(\underline{\boldsymbol{x}}) = V_{SPECIFIED} \end{cases} \tag{2}$$

where $\underline{\boldsymbol{x}}$ represents the material density in each element, $C(\underline{\boldsymbol{x}})$ is the compliance of the structure (a measure of stiffness) and $\boldsymbol{f}$ is the vector of applied loads. The constraints ensure that the structure is in equilibrium with the applied loads, and that the volume of material in the final structure is fixed. This process produces the stiffest possible structure for a given mass of material which satisfies the required boundary conditions, an example of which is given in Fig. 1b (calculated using the program presented by Sigmund, 2001). To minimise the mass of the design, the specified volume of material can be gradually reduced until the required level of compliance is just achieved. Other optimisation techniques such as ESO (*E*volutionary *S*tructural *O*ptimisation, Xie and Steven, 1993) produce similar results in most cases.

Despite offering a powerful means for reducing the weight of products, structural optimisation is usually conducted with no consideration for how the product will be manufactured. Furthermore, it is difficult to impose the geometric constraints which would ease manufacture in this formulation. Consequently, the use of optimisation has to date been confined to high performance applications for instance in aerospace, where the large additional cost of manufacturing is acceptable. Additionally, although the optimisation process produces a lightweight result, it is based on a mathematical optimisation so fails to reveal general principles of lightweight design, so the results of one analysis cannot easily be applied to other products.

Many authors have considered lightweight design without a formal optimisation process, but usually for specific products or classes of product. However, a small number of more general results have been reported. Weaver and Ashby (1997) considered optimal cross-sectional geometries for withstanding different fundamental loadings, e.g. bending or torsion, demonstrating, for example, the optimal cross-sectional shape and size for a tube subjected to a pure torque with strength or stiffness constraints. In a similar manner, Wanner (2010) demonstrated a general process for selecting minimum weight materials for problems where the product volume is constrained. These results provide a useful basis for material and shape selection, but are limited to a set of idealised load cases.

This paper aims to provide a general set of lightweight design principles, not specific to any particular product or application. These principles are then compared to the findings of a set of design case studies, in which five products are assessed to estimate by what fraction their weights could be reduced. Finally, extrapolating these results to all steel and aluminium products, an estimate of the potential for total global requirements for these materials is calculated.

3. Analytical lightweight design of a simple example

This section analyses a simple design problem, in which a point load must be supported above an empty space. This is a typical design challenge: it would be easy to support the load directly by an opposed vertical column, but – for example in an office building, or a car – such a support would intrude on required open space. The example structure is constrained by both strength (the amount of load it can carry before failure), and stiffness (the amount the structure deflects under the applied loading). Several lightweight design principles will be proposed and applied to this example, and the resulting mass of the structure reported.

All calculations are performed using the lengths and forces shown in Fig. 2a and assuming the structure is made from mild steel (σ_y = 275 MPa). In each case, a load of 50 kN must be supported with vertical deflection no greater than 0.028 m (span/360) – the recommended maximum serviceable deflection for floor systems specified in standards for structural steelwork. Full details of the calculations are given in Appendix A.

Fig. 2a demonstrates a frame design, typical of a steel-framed building. The load is carried using a universal beam, supported by two vertical columns; as the load is not aligned with the main structural member, the force is carried by bending in the beam. In Fig. 2b and c a truss structure is used instead, constructed from uniform

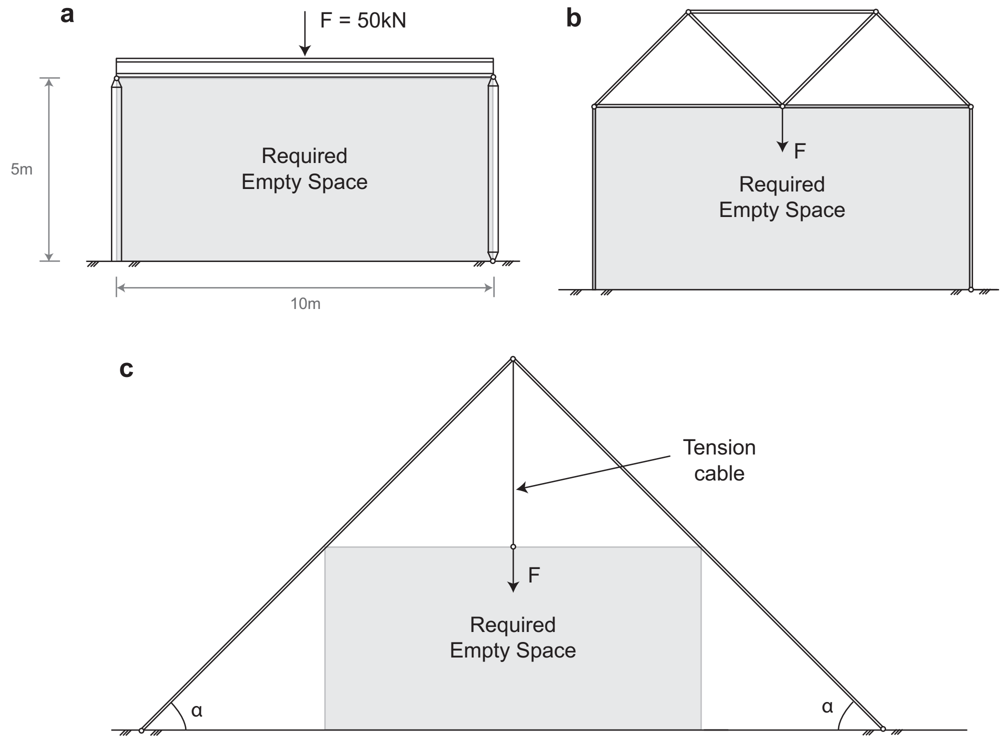


Fig. 2. Structural demonstration of lightweight design principles for (a) conventional I-beam and (b, c) truss structures.

bars with equal cross-sectional areas (for simplicity) and with pin joints so that all loads are aligned with the structural members.

Standard design principles, typical of current commercial practice and based on existing structural design codes, were used to develop an initial design for the three structures, and are given in detail in Appendix A. The first row of Table 1 summarises these designs – providing a reference for likely material use before any optimisation has been performed.

The power of aligning loads with structural members is immediately obvious from the table, as the pin jointed truss structures are both at least 90% lighter than the structure using a standard I-beam, which is loaded in bending.

Each subsequent row of the table shows the weights of the three structures after different lightweight design principles have been applied:

- To demonstrate how over-specification of loads increases product weight, it was assumed that the load had been over-specified by 50% and could be reduced to 33 kN. As shown in the second row of Table 1, this enables weight savings of 34% for both truss structures, and of 15.6% for the frame in Fig. 2a.
- Upgrading the material – for example by new alloying to give increased strength – also offers an opportunity for weight saving. For the truss structures, which are initially constrained by strength, a further weight saving can be gained by upgrading to higher strength steel ($\sigma_y = 355$ MPa), but the design then becomes stiffness constrained, and so the incremental weight saving is limited to around 13%, shown in the third row of Table 1.
- Optimising the components of the design. Calculations in Appendix A allow each bar to have a different area, where for the other designs all bars have the same cross-sectional area. In addition, for the design of Fig. 2c the angle of the main structural members to the ground can be varied, and the appendix shows that the optimal value is 49.4°. More interestingly, for structures loaded in bending such as the beam in Fig. 2a, further weight savings can be made because bending is inherently inefficient: when loaded in tension or compression, a structural member can be fully optimised, because the stress is constant across its area so the minimum required cross-sectional area can be calculated from the specified maximum stress. However, if loads act perpendicular to the member, the stress varies across the cross-section, reaching its maximum only at the greatest distance from the neutral axis. The designer must choose a cross-section such that this maximum value is within the allowed design stress – but therefore all other material in the cross-section is under-utilised. Furthermore, if – as in Fig. 2a – the bending moments vary along the length of the beam, the cross-section of the beam should also vary. The I-beam of Fig. 2a has been optimised using the method described in Appendix B to allow such depth variation, leading to the design shown in Fig. 3. These changes give the final structural weights shown in the last row of Table 1.

Although these results are not exhaustive, and further weight savings may be possible, they demonstrate the power of a few simple design principles in reducing product weights. The difference in weight between the best (26.9 kg) and worst (533 kg) designs is nearly 95%.

From these results a candidate set of principles for lightweight designs can be proposed:

1. Align loads with structural members wherever possible, to avoid bending.
2. Avoid over-specification of loads.
3. Choose the most appropriate material, depending on the limiting performance parameter (e.g. stiffness or strength).

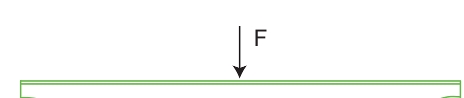


Fig. 3. Variable web depth I-beam, optimised for minimum weight.

Table 1
Summary of weight savings through application of lightweight design principles.

Design	Beam structure		Truss structure A		Truss structure B	
	Mass	Cumulative weight saving	Mass	Cumulative weight saving	Mass	Cumulative weight saving
Baseline	533 kg	–	55.8 kg	–	47.5 kg	–
Minimise overspecification						
Reduce design load to 33 kN	450 kg	15.6%	36.9 kg	33.9%	31.4 kg	33.9%
Material selection						
Material upgrade to 355 MPa	450 kg	15.6%	29.8 kg	46.6%	28.5 kg	40%
Product/component optimisation	275 kg	48.4%	27.9 kg	50%	26.9 kg	43.4%

4. Optimise the final product using the most appropriate method.

In the next section, a set of real world design case studies are analysed to test the extent to which these principles apply in practice, and to investigate the constraints which may prevent their application.

4. Lightweight design case studies

Five case study products have been investigated: universal steel beams, used in construction; steel food cans; car bodies and crash structures; reinforcing bar; and deep sea oil and gas pipeline. Collectively these products account for approximately 300 Mt per year of steel production, which is around 29% of total steel product output in 2008 of 1040 Mt (World Steel Association, 2009). In each case, the general principles of the previous section were applied to demonstrate a potential saving, and this was then reviewed by experts for each case study, to identify constraints or barriers that limit the adoption of the principles.

Here, only the technical potential for reducing metal requirements is assessed. Estimating the impact of these possibilities on metal demand would require a full economic analysis, which is beyond the scope of this paper.

4.1. Universal beams

There are several ways in which structural beams may be overdesigned:

1. Continuous design, in which structural joints are welded and able to transmit moments will produce lighter weight designs than simple design with pin-joints. However, the simple approach is preferred by designers as it is easier to apply, and is also preferred by fabricators and contractors, as the required joints are simpler.
2. The production of standardised sections is convenient for manufacture, but adds weight compared to a section of arbitrary cross-section. At present, to ensure sufficient strength and stiffness, the designer can at best chose a section which exactly meets the requirements. However, more commonly, there will not be a standard section which exactly meets the design requirements, so a stronger or stiffer beam than required must be used.
3. Floor and roof beams are usually subject to transverse loads, so have a varying moment distribution. They should therefore not be prismatic, but have a cross-section that varies with the bending moment along the beam. Pedersen and Pedersen (2009) demonstrated this analytically for a variety of load conditions and cross-sectional shapes. Varying cross-section beams are, however, difficult to manufacture, and complicate construction.

To investigate weight saving potential in structural beams, a set of design studies were conducted. Three sample load cases were used, two of which are typical of the load magnitude and span of beams in floor systems, and a third typical of the loads commonly encountered in roof systems. The load cases are shown schematically in Fig. 4.

Several different designs of beam were then considered, and the weights of the beams compared. As a baseline case for floor beams, a standard universal beam was used, against which beam weight savings are calculated. In reality, composite floor systems (described below) are increasingly common, and anecdotal evidence suggests that they may be used in up to 50% of new building projects in the UK. Some of the beam designs considered here, such as the variable cross-section beams, could also be used as part of a composite floor system, thereby compounding the weight saving over a non-composite standard beam. For the roof beam, a standard universal beam was taken as the baseline case.

The beam designs (shown in Fig. 5) were:

1. A standard universal beam.
2. A composite floor beam, comprising a standard beam which is joined to a concrete floor slab. The concrete floor slab carriers some of the compressive bending stress, so a smaller universal beam can be used compared to a beam used in isolation.
3. A variable web depth beam – an I-section beam, with a varying web depth along the length.
4. A variable flange width beam – an I-section beam, with a varying flange width along the length.
5. A cellular beam – a standard beam with “cells” cut from the web.
6. A Fabsec™ beam – manufactured by welding steel plates to webs which are cut from sheet.
7. An open-web joist.
8. A cold-formed section, e.g. a Z-section.

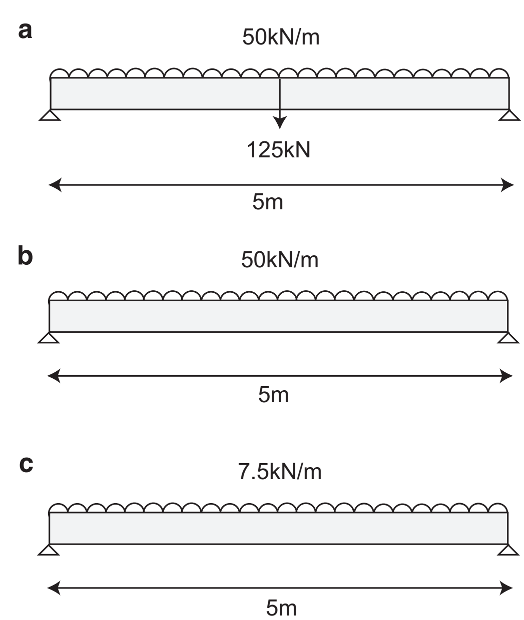


Fig. 4. Load cases for beam case study.

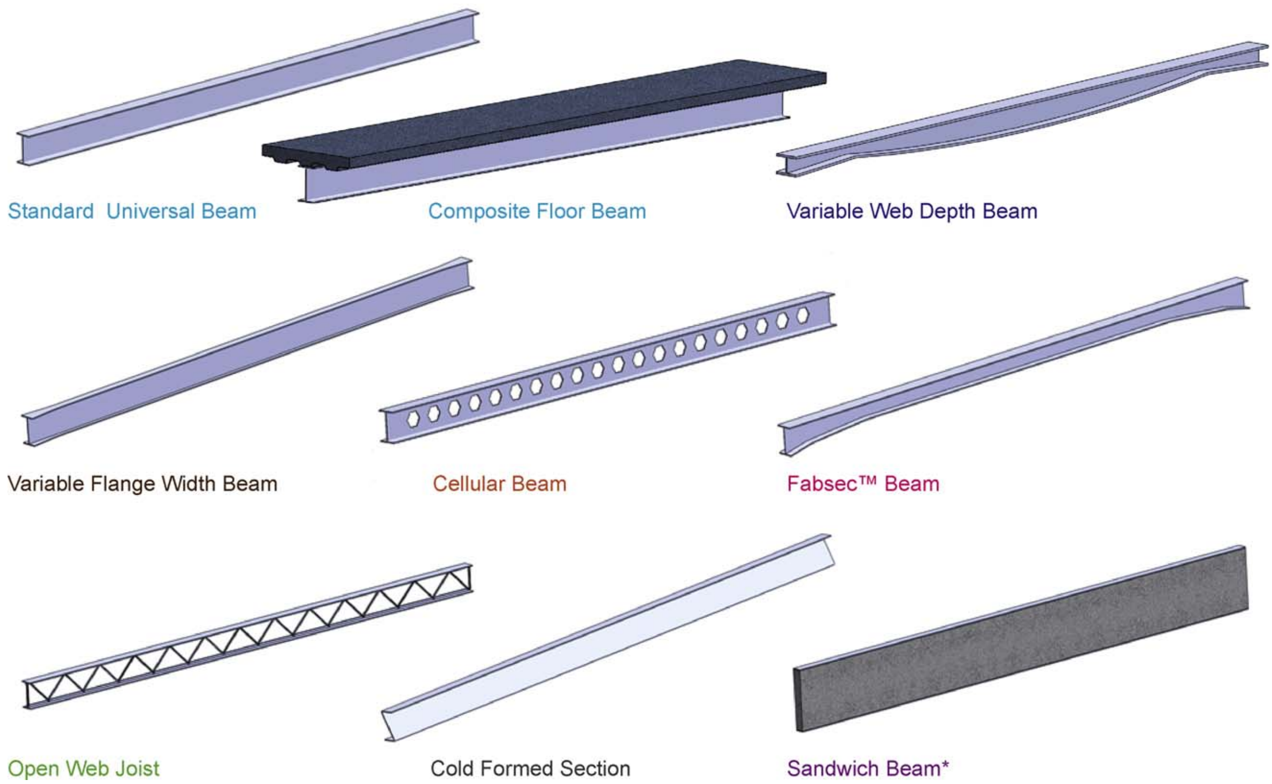


Fig. 5. Alternative beam designs.

9. A sandwich beam.

In load cases 1 and 2, it is assumed that the dead load is 14 kN m^{-1} and the imposed load is 19 kN m^{-1}, giving a ULS load of 50 kN m^{-1}. Load case 1, representative of a primary floor beam, which supports another beam at its centre, also has a 125 kN point load applied at the centre of the span. Load case 3 is scaled linearly to produce a ULS load of 7.5 kN m^{-1}.

Where possible, the beams are sized according to guidance in appropriate standards or design guides. Morris and Plum (1996) describe the design process for standard universal beams, Vulcraft (2007) provide a catalogue of open web joists with suitable load magnitudes, Bluescope (2010) produce a similar catalogue for cold-formed sections, and Allen (1969) describes the design of sandwich beams. However, the beams with variable cross-sectional geometry along their length are novel, so the approach of Appendix B was used. For the roof beam, which is not restrained against lateral displacement, lateral torsional buckling must also be considered, as described by Morris and Plum (1996). Due to the complexity of calculating lateral torsional buckling performance for non-standard (and non-prismatic) sections, not all the designs were used for the roof beam case study (load case 3). However, the large weight savings demonstrated by the open web joist and cold formed section are unlikely to be exceeded by other designs.

For each of the three load cases, a design was sought for each of the beam types shown in Fig. 5, and the weights of these designs were compared to the benchmark cases (described above). In some cases, such as the open-web joist in load cases 1 and 2, no feasible design could be found. The results of these case studies, shown in Fig. 6, demonstrate that around 30% of the weight of a standard beam could be saved through the use of optimised, varying cross-section I-beams. Other alternative designs such as cellular beams and sandwich beams also offered some weight saving. These case studies were conducted using the 'simple' design approach, using the procedures and recommended serviceability requirements in the British Standards for structural steelwork (BS-5950). If a continuous design were implemented, then further weight savings might be possible.

An important question remains as to whether optimised beam designs are less robust than standardised alternatives. Providing infrequent but severe loads (e.g. from hurricanes) are factored into the design load case, an optimised beam will remain safe, but a standardised beam may provide additional protection against extreme loads which were not anticipated at the design stage. In the face of climate uncertainty over the next few decades, this may be desirable. An open question remains, however, as to what extent energy intensive materials should be used as a hedge against unknown in-service loads.

4.2. Food cans

Around 100 billion food cans are produced each year, mainly from steel. Over the past 20 years, *beverage* cans have seen large reductions in weight, but similar reductions have not been observed in the weight of *food* cans. This is largely because the design of these cans is limited not by the design requirements of food transportation, but by supply chain requirements before the product reaches the end-user.

After manufacture, cans are sent to food manufacturers where they are filled and capped. The cans are then cooked in a process known as 'retorting', where they are subjected to a large net external pressure (~1.0 bar) then later in the process a large net internal pressure (~+3.0 bar). The minimum allowable material thickness of both the ends and walls of the cans are influenced by the requirement that they be undamaged during retorting. Further along the supply chain, the cans are placed on pallets and stacked, without any additional support, and the cans must be capable of supporting up to 50 times their own weight. This performance requirement

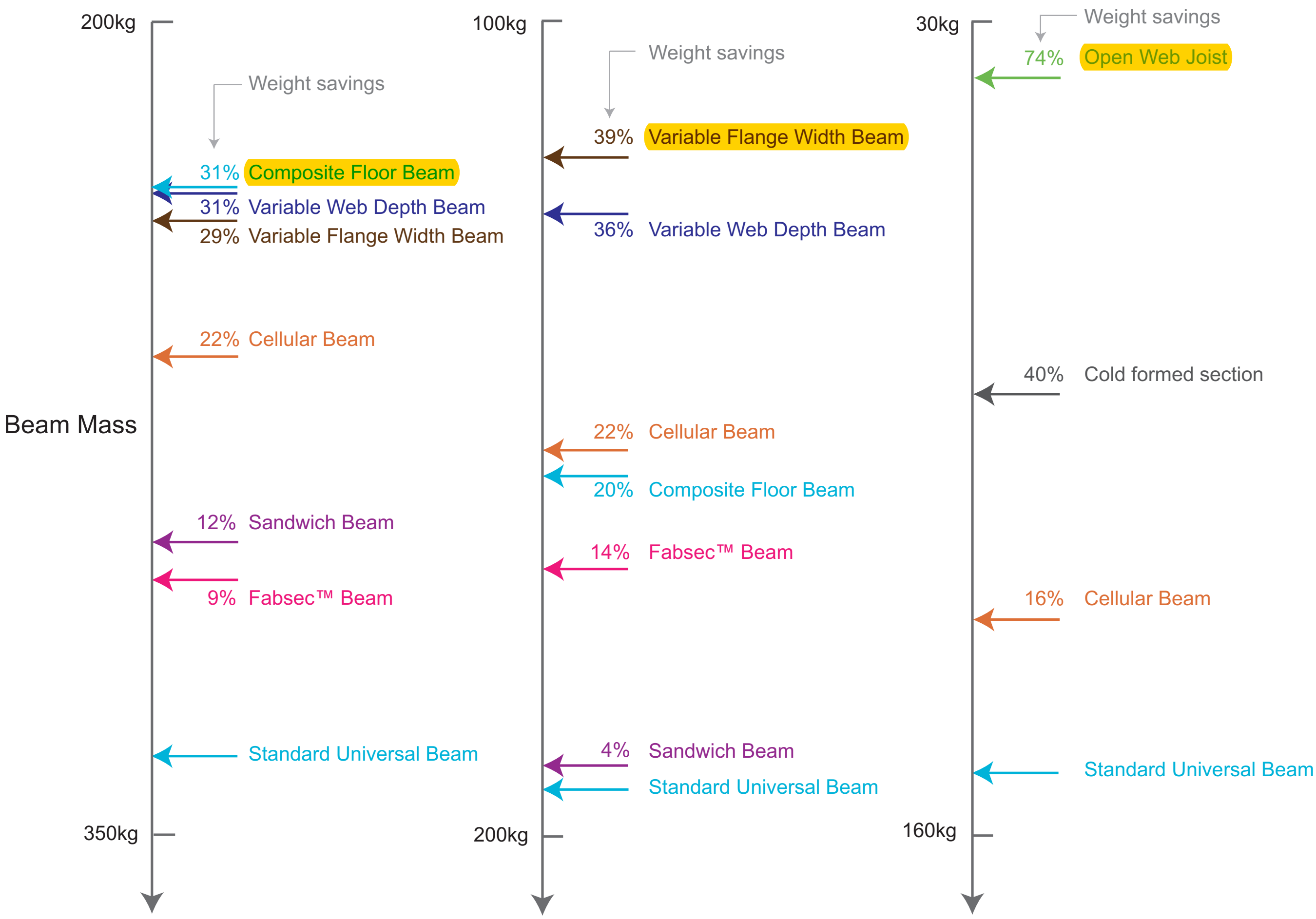


Fig. 6. Weight saving summaries from load cases 1–3.

under axial crushing further restricts the minimum weight of the can.

The treatment of food cans is in stark contrast to the handling of other types of packaging, such as Tetrapak™ and aluminium pouches. Food packaged in this manner is subjected to a more advanced retorting process, which generates much milder pressures (−0.5 bar to +0.5 bar), and they are stacked in secondary packaging which supports the weight of the pallets above. Food cans could be treated in the same way but the supply chain currently operates on the assumption of previous robustness. The transition to using lighter cans would require simultaneous change by multiple companies at multiple points along the supply chain and significant capital investment.

Carruth (2011) analysed the whole supply chain for steel food cans in order to identify possibilities for weight savings, and reported that if food cans were treated in the same way as other packaging, the weight of the can bodies could be reduced by around 30%, and the can ends by a similar amount, and potentially more if foil closures were used (though these may not be suitable for all products). In total, therefore, a weight saving of at least 30% is achievable simply through the use of thinner material and existing manufacturing processes. However, co-ordinating this change along the supply chain would be difficult, and economic factors may make these changes unattractive.

4.3. Car bodies

Car bodies serve two major functions: to provide a base structure to which other parts of the car will be connected; and to protect the occupants in the event of a collision through the use of crash elements. The design and manufacture of car bodies requires a careful balance between material selection, manufacturability and performance.

Given the complexity in the design of a car body and the open-ended nature of the design problem, estimating the light-weighting potential for car bodies as a whole is difficult. However, by looking at the fundamental performance requirements of the car body, particularly the crash structures, and surveying the available literature, a rough estimate can be made.

Crash elements are required to dissipate energy during impact. Both the design of the element and its material are important. For a lightweight crash structure, a material with high specific energy absorption (energy absorbed per unit mass) should be used. Fig. 7, taken from USDTRITA (2008), compares the specific energy absorption of a variety of materials. It can be seen that CFRP performs significantly better than other materials. Although ultra high strength steels and aluminium alloys are continuously being developed to improve their specific energy absorption, these results suggest that a CFRP crash element may be several times lighter than an equivalent steel or aluminium element. As stated previously, the design of the element is also important. Kim (2002) demonstrates an optimisation procedure for a thin-walled front crash element, and demonstrates that the weight of a standard square hollow tube crash member can be reduced by nearly 50% by optimising the design. Combining CFRP materials with optimised designs, therefore, may produce weight savings in excess of 90%.

However, when considering material substitution, the CO_2 emissions and embodied energy associated with production and

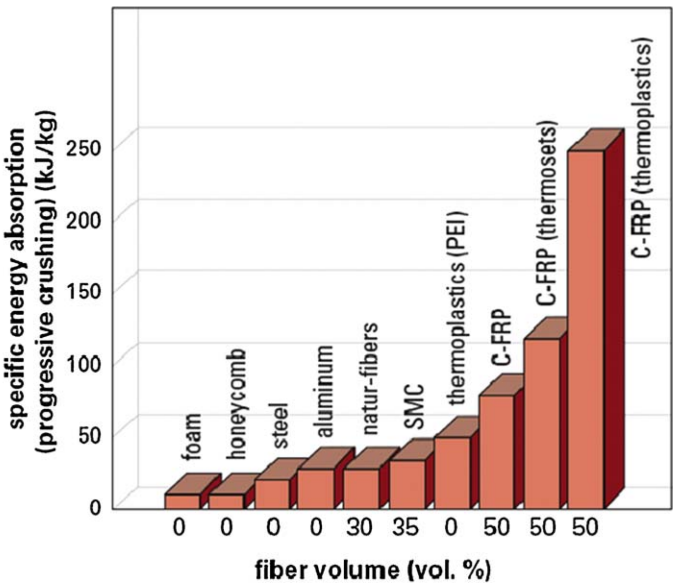


Fig. 7. Comparison of specific energy absorption of different materials. From USDTRITA (2008).

manufacture must be considered. The production of CFRP has significantly higher emissions than steel and aluminium. If CO_2 emissions reduction is the primary aim of the design change, these higher emissions must be offset by a reduction in use-phase emissions for material substitution to be a benefit. As outlined in the introduction to this paper, to simplify the analysis, here weight saving opportunities through substitution with a new material are not considered. However, weight saving by alloy upgrade within the same material (e.g. steel replaced with another steel of higher strength) will be considered, along with weight savings possible through design optimisation.

Crash elements comprise only a relatively small amount of the overall car body weight. Weight saving possibilities for the rest of the car body are summarised by Cheah (2010), who reports that the targeted weight savings of major car manufacturers (such as Ford and GM), range from 10% to 25% over the next 5–10 years. The strategies to be used in meeting these targets are not clear. However, Lotus (2010) give a detailed breakdown of light-weighting opportunities for an example car body to be released around 2017–2020. Of the opportunities described, those involving “low development” alloy upgrade offered weight savings of around 17.5%, by replacing existing steel parts with higher strength steels, and “high development” (i.e. greater technical complexity and cost) opportunities offered savings of around 25%.

In summary, it is clear that there is great potential for weight saving in car body structures through design improvement and material substitution, both with higher performance alloys of the same material, and with alternative materials. However, in order to report possible demand reduction opportunities for steel and aluminium accurately, only those opportunities involving design improvements or alloy substitutions are considered, which are estimated to offer weight savings of 17.5–25%. These estimates are in line with the targets reported by major car manufacturers, reported by Cheah (2010).

4.4. Reinforcing bar (rebar)

Steel bar, used to provide reinforcement in concrete structures, is the single largest end user of steel products, accounting for around 170 Mt per year (just over 10% of all steel output), 60% of which is used in China. The reinforcement is used to provide tensional strength in concrete structures, and also to prevent cracks. It is supplied in different grades, depending on the yield strength of the material, and sometimes with different levels of ductility, though this varies from country to country.

A large proportion of rebar is constrained by strength, so the lightweight principle of material selection would suggest that the strongest possible material should be used, provided ductility requirements are satisfied. Additionally, to ease fabrication and onsite construction, rebar is often used in a sub-optimal manner, with the same bar sizes and spacing used over large areas, where a more efficient design would vary both. What fraction of the material being used as reinforcement could be saved through the use of stronger material, used in an optimised manner?

As China is the dominant user of reinforcing steel, this section focuses on standard Chinese design practices. In China, the majority of rebar in use is of 335 MPa grade, with higher strength used for around 40% of the market (Caifu, 2010). Caifu (2010) reports that a 14% weight saving can be achieved by upgrading from 335 MPa to 400 MPa rebar, and a further 10% saving can be achieved by upgrading to 500 MPa rebar. In the rest of the world, the use of 400 MPa rebar is standard, with some 500 MPa rebar also in use, which has equivalent or higher levels of ductility. Higher strength rebar is available, and may enable further weight savings, but its high cost currently limits its use in all but the most critical applications (for example in earthquake regions). The weight savings from optimal bar selection and layout are harder to estimate. Moynihan (2011) estimates that 10–15% of the total rebar weight can be saved with optimised systems, but only for certain types of reinforcement.

Outside China, the use of high strength rebar is already common, so small further weight savings could be made by using higher strength rebar >500 MPa, but this is rarely economical. In China, upgrading all rebar to 500 MPa strength would save 23 Mt/year of steel (Allwood et al., 2011). Further it is assumed that globally, optimised rebar systems could be used in 65% of building projects and 50% of infrastructure projects, which would save a further 28 Mt/year. Therefore, in total, material selection and product optimisation could save 51 Mt of steel per year, equivalent to 30% of all rebar production.

4.5. Deep sea oil and gas pipeline

Deep-sea oil and gas pipeline is made from high grade steel, operating at depths of over 2 km. The steel is both significantly stronger and more corrosion resistant than conventional mild steel, and accounts for around 25 Mt/year of steel production.

In service, the pipeline is required to carry high pressure oil and gas over long distances in harsh environments. The pressure of the oil/gas is usually similar to the external hydrostatic water pressure, resulting in a low overall load on the pipe. However, the minimum weight of the pipe is limited by failsafe requirements in service (specifically that the pipe does not yield or buckle under loss of internal pressure) and by pre-service installation requirements. In this case study, savings estimates are made based on a real world example described by Pulici et al. (2003).

The installation system for deep sea pipeline places high loads on the pipe itself. In deep waters (>2 km), pipes are laid using a system known as ‘J-lay’, illustrated graphically in Fig. 8. The pipe “string” is hung from a laying barge and allowed to fall to the sea floor. As the barge moves forward, new sections of pipe are welded to the end of the empty pipe string, which is then lowered further into the water. This generates high stresses due to the large external water pressure acting on the empty, unpressurised pipe, and due to the bending of the pipe in the ‘sagbend’ region.

If the pipe could be pressurised internally during laying to negate the effect of the external water pressure, the pipe walls could be made thinner. One means of doing this would be to flood the pipe

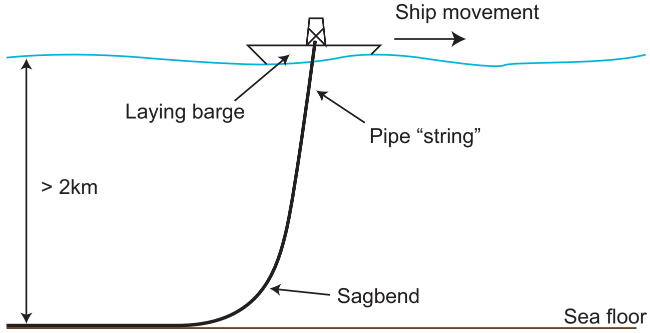


Fig. 8. J-lay pipe installation system.

with seawater during installation. However, flooding the pipe also makes the pipe significantly heavier due to the loss of buoyancy forces which are present only if the pipe is empty and this would sink the laying barge.

An alternative means of pressurisation would be to pump high pressure gas into the pipe, allowing the wall thickness to be reduced. The amount of thickness reduction depends on the amount of corrosion protection which is required, as the pipe walls are often thickened to provide a certain lifetime against corrosive failure. Taking these corrosion requirements into account gives a possible wall thickness reduction of 10–30%.

The upper 30% limit is governed by failsafe requirements in service. Under a complete loss of pipe pressure in service, the pipe must not yield or buckle. The wall thickness required to prevent either of these from occurring is shown in Fig. 9, for a case where the original wall thickness was 0.0318 m. The minimum acceptable wall thickness occurs at the crossover point of the failsafe pressure and the actual collapse pressure, giving a weight decrease of around 30%.

Providing a high volume of high pressure gas would be difficult, and have serious safety implications during installation. This makes it an unlikely candidate for weight reduction, although is a technical possibility. Upgrading pipe material to higher strength may also offer some weight saving, but with the use of high strength (≥550 MPa) steels already standard, this may not have much further potential.

5. General design principles and constraints of lightweight design

Using the analytical example from Section 3, and the lessons learnt from the design case studies described in Section 4, here a final set of general principles are presented for lightweight design, followed by a set of constraints which may limit their application.

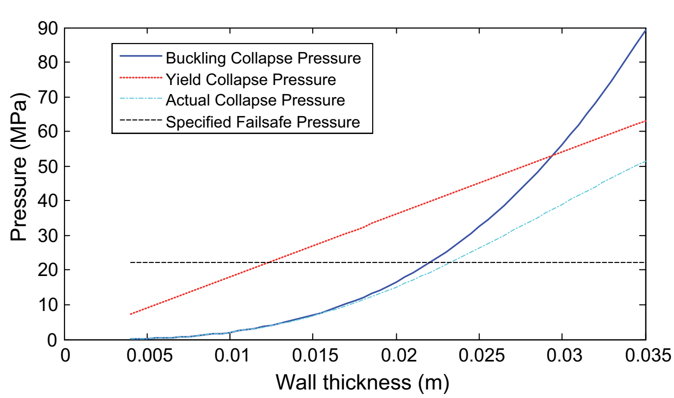


Fig. 9. Wall thickness requirements to prevent buckling and yield.

- Minimise over-specification of loads
  Over-specification can occur if the level of in-service loading is unknown, or because the end use of the product is not known with certainty at the design stage. This occurs, for example in structural design with “speculative” development projects, when buildings are constructed before a client has been identified to use the building. The cost of including additional (and potentially redundant) material in the initial design is significantly less than retrospectively upgrading the structure of the building if its design load turns out to be insufficient. Therefore high design loads may be specified which are significantly greater than any load encountered during final service.
- Align components with loads
  Loads are carried more efficiently when they are aligned with the structural members supporting them. Elimination of bending from structures may be difficult, but is an important goal.
- Material selection
  Material selection is not straightforward due to the interaction of requirements such as strength, stiffness, specific energy absorption, or other material parameters. In some cases material substitution can offer no weight saving benefit. For example, if a design is stiffness constrained and restricted to a particular material, e.g. steel, then upgrading material properties may be of no benefit, as alloying has little effect on material stiffness.
- Product-level optimisation
  Some opportunities for optimisation occur only at the early stages of product design – for instance, having a single structural frame supporting all loads, will always be more efficient than having independent frames for different loading groups. This was exploited at the London Olympics Velodrome, where the seating is mounted on the columns that support the roof, rather than having independent seating supports as is conventional.
- Component-level optimisation
  Several approaches to component optimisation have been discussed in this paper, but should account for manufacturing requirements as well as final product requirements. Reducing the weight of a component by 30% saves no material if an additional 30% is scrapped during manufacture – as is common in aircraft production.

These principles cannot always be applied fully, due to other constraints. A common constraint is cost: lighter weight products tend to be more complicated geometrically, and hence more expensive to manufacture, and switching to a new manufacturing route may require significant capital investment in new equipment. However, the list below covers only technical constraints. Moving down the supply chain from designer to end-user, the key constraints identified in the product case studies were:

- Design specification and risk
  Often the cost of including additional material in a design is negligible compared to the risk of component failure, and this drives heavier design. Furthermore, the actual loads to be encountered in-service may not be well understood, and hence over-specified. Conservatism within design codes, for example serviceability recommendations (i.e. maximum deflection) may also drive over-specification. This constraint is particularly relevant to the deep-sea pipeline case study, where the consequences of product failure could be catastrophic.
- Pre-use service requirements
  In some cases, product design is limited by performance requirements *before* the product reaches service. This may be due to high loads applied during installation, for example, or

Table 2
Map of constraints and technical principals found in product case studies.

	Technical principles				Constraints				
	Avoid overspec.	Material selection	Product optimisation	Component optimisation	Design spec. and risk	Pre-use service	Manufacturing route	Consumer perception	End-of-life re-use
Universal beams	◦			•	◦		•		◦
Food cans	•					•			
Car bodies		•	•	•			◦	•	
Reinforcing bar		•	•						
Linepipe	•				•	•			

(•) Applicable; (◦) might be applicable.

because the product is subjected to processing which places significant demands on the product itself. In some applications, this means that the product is significantly heavier than its *in-service* performance requires. This constraint was demonstrated in both the deep-sea pipeline and food can case studies described in the previous section, where pre-use service requirements dictated the weight of the final product. In many cases it is possible to find alternative temporary means to carry loads pre-service to avoid this requirement for increased weight.

- Manufacturing constraints

  A major constraint in designing lightweight products is the difficulty of finding a cost-effective manufacturing route. For example, hot rolling processes are currently restricted to the creation of prismatic structural beams, whereas optimised beams should generally have variable cross-section. Typically, optimised components have geometric variations which cannot be created with traditional manufacturing processes which have been optimised for economies of scale. However, the development of novel, flexible forming processes, such as the process for rolling variable cross-section beams presented by Carruth and Allwood (2011), offer the possibility for economical manufacturing of lighter weight products.
- Negative consumer perceptions

  Lightweight products may be perceived as inferior by end-users. Lightweight car body panels, for example, such as those examined in the previous section, may deflect when pressed, giving the impression of being inferior, even if they perform the same function as a heavier alternative and are technically sound.
- Options at end-of-life

  An important future strategy for reducing material demand is to re-use products at the end of their life (see Allwood et al. (2010) for an overview). If new products are optimised for a specific purpose, it may be more difficult to re-use them at end-of-life than standardised alternatives. This is particularly relevant to the universal beam case study, where the optimisation of beams for specific load cases and spans might inhibit re-use at end of life.

Awareness of these constraints may mean that design briefs, or in-service product requirements could be adapted to promote light-weighting. Table 2 shows how the general design principles and constraints described in this section relate to the case studies from the previous section.

6. Estimating the scale of the global opportunity for lightweight design

Having conducted five case studies, this section aims to extrapolate from them to predict the overall potential for lightweight design to reduce global requirements for steel and aluminium. The results of the individual case studies are summarised in Fig. 10b, and the effect of these savings on overall steel requirements is shown in Fig. 10a, for both a best case (assuming that actual weight savings are at the upper end of their estimated ranges) and a worst case (that weight savings are at the lower end of their ranges). It can be seen that, in general, weight savings of 25–30% are possible (though very difficult in the case of deep sea pipeline). If a weight saving of 25% applied only across these 5 products, annual steel requirements would be reduced by 75 Mt, or around 7% of total global production of steel products. For aluminium, the estimate is harder to calculate due to the different breakdown of products compared to steel. However, 25% weight savings for cars and structural aluminium products would save 3.25 Mt per year, around 7% of total annual aluminium product output.

In all the case studies considered here, savings of 25–30% were possible in the best case, although in some cases difficult or costly to implement. If these savings could be applied universally across

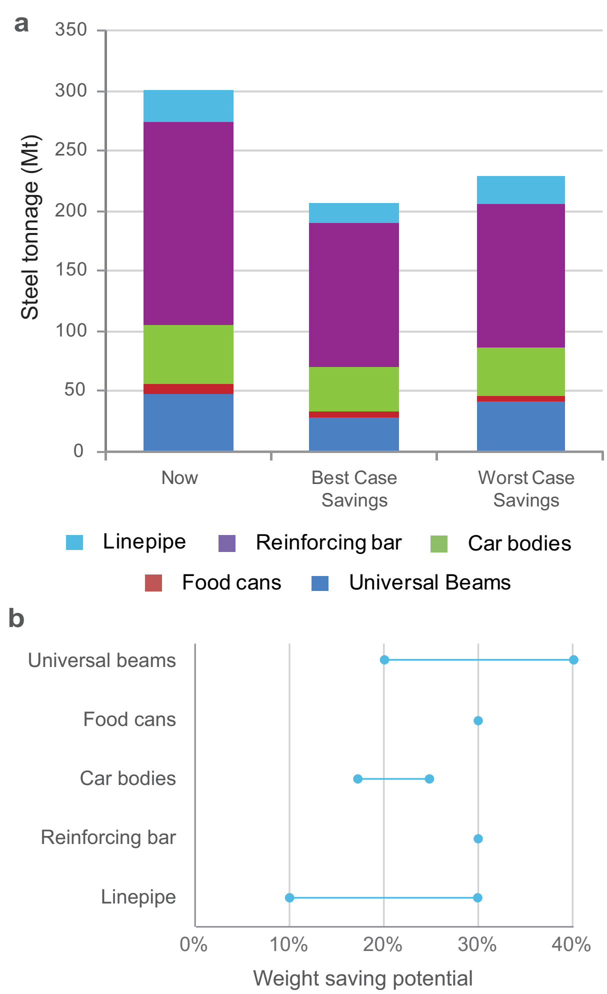


Fig. 10. Summary of potential reduction in material requirements in design case studies.

all steel and aluminium products, then yearly production could be reduced by 350–400 Mt for steel and 11–15 Mt for aluminium.

7. Discussion and conclusions

Lightweight design has potential as a strategy for CO_2 emissions abatement as it enables a reduction in total primary material production (i.e. from ore). Other material efficiency strategies such as improving yield ratios along the supply chain, although beneficial, only lead to reductions in the level of secondary material production (i.e. from scrap), which is less energy and carbon intensive than primary production. There are also additional use-phase emissions benefits from light-weight design, particularly for vehicles, where each kilogram of weight saving produces an emissions saving of around 10,000 kg CO_2 for aircraft and 10 kg CO_2 for cars over their lifetimes.

Many of the light-weighting measures discussed in this paper are, at present, economically unattractive. However, economic incentives to reduce carbon emissions may make the measures described in this paper more attractive in a wider range of cases.

Acknowledgement

This paper is an output of the WellMet2050 project (www.wellmet2050.com), supported by a £1.4 m Leadership Fellowship provided by the UK Engineering and Physical Sciences Research Council (EPSRC) reference EP/G007217/1.

Appendix A. Calculations for lightweight design example

The initial designs for the example in Section 3 are calculated using well established techniques. The size of the standard universal beam is calculated to provide sufficient plastic moment capacity and stiffness, in order to meet the maximum deflection requirements. The following formulae are used:

$$M_{max} = (-)\frac{FL}{4} \tag{A.1}$$

$$\delta_{max} = \frac{FL^3}{48EI} \tag{A.2}$$

Using values of F = 50 kN, L = 5 m and δ_{max} = 0.028 m, the following beam requirements are calculated:

$$Z_p = 454\,\text{cm}^3, \quad I = 17,857\,\text{cm}^4 \tag{A.3}$$

Comparing these requirements to the beams available in the standard section tables, the lightest suitable beam is a 406 × 140 × 53 UB, with a weight of 53.3 kg/m, giving a total weight of 533 kg. If the load is reduced to 33 kN, then using the same formulae as above, the 356 × 171 × 35 UB is the lightest suitable beam, with a total weight of 450 kg.

The truss structures are sized according to strength and stiffness constraints. Using the bar numbering scheme shown in Fig. A.1, the bar tensions t_i are calculated from simple equilibrium applied horizontally and vertically at each pin joint, and hence the bar extensions can be calculated using the bar areas A_i, bar lengths L_i and Young's modulus E (210 GPa). These results are summarised in Tables A.1 and A.2 for trusses A and B.

The first three designs for each truss structure use the same cross-sectional bar areas for each bar. In the final optimisation stage, the bar areas are varied individually to produce the least weight solution, using the built-in numerical solver in Microsoft Excel 2007. In truss structure B, the angle of the bars to the ground is also varied to produce an optimal value of 49.4°, having assumed a value of 45° in the initial designs.

Appendix B. Design of variable cross-section I-beams

Universal I-beams are used throughout steel-framed buildings to support floors and roofs. They are produced in a set of standard sizes as prismatic sections (i.e. having constant cross-section along their lengths). Their size is chosen to provide sufficient strength and stiffness based on an assumed design load, guidelines for which are published as part of the Eurocode 3 standard for structural steelwork. Typically the design is conducted using a "simple" design philosophy, which assumes that no loads are transferred to the ends of the beam through structural columns. The alternative, "continuous" design, allows for the transfer of load between beams, but is considerably more complicated, and requires the use of computerised design software.

The design of I-beams with varying cross-section is not covered by current building standards, so in this section a design is developed building on elements of existing design codes. The simple design methodology will be used, and the necessary checks for strength and stability performed at all points along the beam. The simple design methodology uses the concept of limit states to specify the critical load acting on a structural member. In limit state design, the ultimate limit state (ULS) is the load at which the beam is assumed to fail completely. In the examples considered above, the ULS imposed loads are as shown in Fig. 2a.

The stiffness requirements of the beam are governed by the "serviceability limit state", which specifies the maximum recommended deflection at the centre of the beam when subjected to the imposed load. For floor beams (used in this example) this should satisfy:

$$\delta < \frac{L}{360} \tag{B.1}$$

and for roof beams:

$$\delta < \frac{L}{200} \tag{B.2}$$

where L is the beam span and δ is the maximum deflection.

The structural beam must be sized to provide sufficient shear and moment capacity at all points along its length. The shear capacity of the beam is given by the expression:

$$P_v = 0.6p_yA_v = 0.6p_ytD \tag{B.3}$$

where p_y is the yield strength (assumed to be 275 MPa, or 265 MPa if the thickness of any part of the section exceeds 16 mm), t is the web thickness, and D is the web depth. If the shear force in the beam is less than $0.6P_v$, it is considered to be in a "low shear" state, and the full moment capacity of the beam can be used. If the shear force exceeds this value, then the moment capacity must be reduced. In this case study, members are always sized such that the beam is in the low shear state.

The moment capacity of the beam is given by:

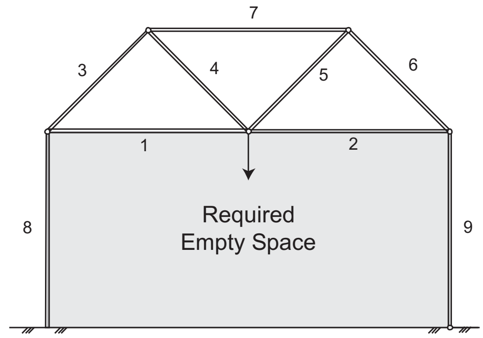
$$M_c = p_yS_{xx} \tag{B.4}$$

but with the restriction that:

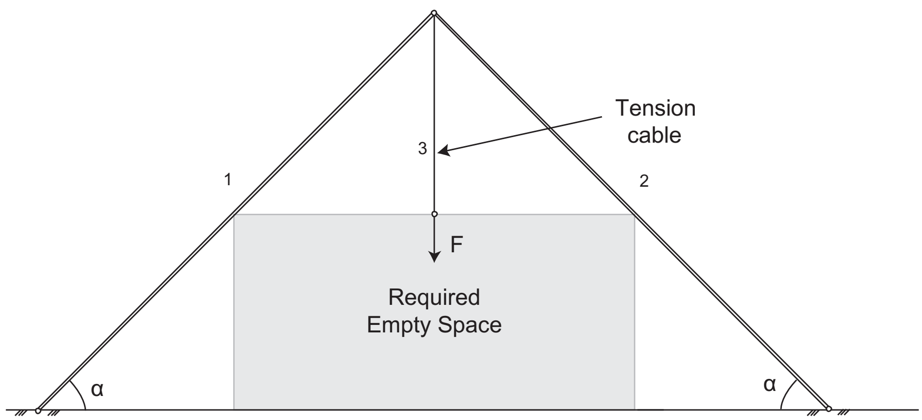
$$M_C \not> 1.2p_yZ_{xx} \tag{B.5}$$

where S_{xx} and Z_{xx} are plastic and elastic section moduli respectively. The second expression ensures that the beam does not become plastic at any point under normal working loads.

A restriction must be placed on the aspect ratio of the flange and web to ensure that the beam will not buckle locally before reaching the ultimate limit state, allowing the section to be considered "class 1 plastic", meaning that the full plastic capacity of the beam can be



(a)



(b)

Fig. A.1. Bar numbering scheme for truss structures A and B.

utilised. For an I-section, these restrictions are given by (using the dimensions shown in Fig. B.1):

$$\frac{B}{2T} < 9\varepsilon \tag{B.6}$$

$$\frac{d}{t} < 80\varepsilon \tag{B.7}$$

where

$$\varepsilon = \left(\frac{275}{p_y}\right)^{0.5} \tag{B.8}$$

To design the variable web depth members, the beams discretised along their length, with a depth variable being specified at each discrete point. These variables, along with the other cross-sectional dimensions (B, T, t in Fig. B.1) are supplied to an optimisation routine, which seeks to minimise the objective function:

$$f = m + \delta_{penalty} \tag{B.9}$$

where $\delta_{penalty}$ is a variable designed to penalise unacceptably large displacement, defined as:

$$\delta_{penalty} = \begin{cases} 0, & \delta_{actual} < \delta_{max} \\ (\delta_{actual} - \delta_{max})^2 \times 10^4, & \delta_{actual} > \delta_{max} \end{cases} \tag{B.10}$$

The mass of the beam is calculated by numerically integrating the expression:

$$m = \int_0^L A(x)dx \tag{B.11}$$

The deflection of the beam is calculated using a form of the virtual work equation:

$$\sum F^*\delta + \int_0^L w^*(s)\delta(s)ds + \sum C^*\theta = \int_0^L M^*(s)\kappa(s)ds \tag{B.12}$$

where the forces/moments marked with * represent the virtual equilibrium system, and the other expressions represent the real displacements, rotations and curvatures. To find the displacement at the centre of the beam, the virtual force system consists of a point force of unit magnitude at the centre of the beam, with associated virtual bending moment $M^*(x) = x/2$, for $x < L/2$. Using the

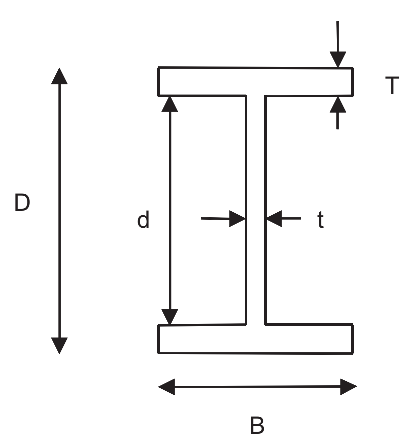


Fig. B.1. I-section dimensions.

Table A.1
Full calculation details for truss structure A.

Bar	L_i (m)	Initial				Reduced load				Material upgrade				Optimised			
		t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)	t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)	t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)	t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)
1	5	25	181	3.3	138	16.5	120	3.3	138	16.5	97	4.1	170	16.5	71	5.5	233
2	5	25	181	3.3	138	16.5	120	3.3	138	16.5	97	4.1	170	16.5	71	5.5	233
3	3.54	−35.3	181	−3.3	−195	−23.3	120	−3.3	−195	−23.3	97	−4.1	−241	−23.3	100	−3.9	−233
4	3.54	35.3	181	3.3	195	23.3	120	3.3	195	23.3	97	4.1	241	23.3	100	3.9	233
5	3.54	35.3	181	3.3	195	23.3	120	3.3	195	23.3	97	4.1	241	23.3	100	3.9	233
6	3.54	−35.3	181	−3.3	−195	−23.3	120	−3.3	−195	−23.3	97	−4.1	−241	−23.3	100	−3.9	−233
7	5	−50	181	−6.6	−275	−33	120	−6.6	−275	−33	97	−8.1	−340	−33	141	−5.5	−233
8	5	−25	181	−3.3	−138	−16.5	120	−3.3	−138	−16.5	97	−4.1	−170	−16.5	71	−5.5	233
9	5	−25	181	−3.3	−138	−16.5	120	−3.3	−138	−16.5	97	−4.1	−170	−16.5	71	−5.5	233
		$m=\sum \rho A_i L_i = 55.8$ kg				$m=\sum \rho A_i L_i = 36.9$ kg				$m=\sum \rho A_i L_i = 29.8$ kg				$m=\sum \rho A_i L_i = 27.9$ kg			
		$\delta = \frac{1}{F}\sum t_i e_i = 14.8$ mm				$\delta = \frac{1}{F}\sum t_i e_i = 22.4$ mm				$\delta = \frac{1}{F}\sum t_i e_i = 27.7$ mm				$\delta = \frac{1}{F}\sum t_i e_i = 27.7$ mm			

Table A.2
Full calculation details for truss structure B.

Bar	L_i (m)	Initial				Reduced load				Material upgrade				Optimised				
		t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)	t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)	t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)	L_i (m)	t_i (kN)	A_i (mm^2)	e_i (mm)	σ_i (MPa)
1	14.1	−35.3	182	−13.1	−181	−21.7	120	−13.1	−181	−21.7	103	−14.3	−217	14.3	−21.7	92	−16.1	−237
2	14.1	−35.3	182	−13.1	−181	−21.7	120	−13.1	−181	−21.7	103	−14.3	−217	14.3	−21.7	92	−16.1	−237
3	5	50	182	6.6	275	33	120	6.6	275	33	103	8.9	307	5	33	139	6.6	237
		$m=\sum \rho A_i L_i = 47.5$ kg				$m=\sum \rho A_i L_i = 31.4$ kg				$m=\sum \rho A_i L_i = 28.5$ kg				$m=\sum \rho A_i L_i = 26.9$ kg				
		$\delta = \frac{1}{F}\sum t_i e_i = 25.0$ mm				$\delta = \frac{1}{F}\sum t_i e_i = 25.0$ mm				$\delta = \frac{1}{F}\sum t_i e_i = 27.7$ mm				$\delta = 1/F\sum t_i e_i = 27.7$ mm				
		$\alpha = 45^\circ$				$\alpha = 45^\circ$				$\alpha = 45^\circ$				$\alpha = 49.4^\circ$				

Euler beam bending formula for curvature, this expression can be re-written as:

$$\delta_{actual} = \frac{2}{E}\int_0^{L/2} \left(\frac{x}{2}\right)\frac{M(x)}{I(x)}\ dx \tag{B.13}$$

where δ is the displacement at the centre of the beam, $M(x)$ is the real bending moment at location x (measured from the left hand end of the beam), and $I(x)$ is the second moment of area at location x. This expression is calculated by numerical integration along the length of the beam.

The optimisation procedure is constrained to ensure that sufficient moment capacity and shear capacity is provided at each location along the beam, and that the local buckling criterion for the web and flange slenderness ratios (described previously) are observed.

References

Allen HG. Analysis and design of structural sandwich panels. Oxford: Pergamon Press; 1969.

Allwood JM, Cullen JM, Carruth MA, Cooper DR, McBrien M, Milford RL, et al. Sustainable materials with both eyes open. Cambridge, UK: UIT; 2012.

Allwood JM, Cullen JM, Carruth MA, Milford RL, Patel ACH, Moynihan M, et al. Going on a metal diet. University of Cambridge; 2011, ISBN 978-0-903428-32-3, Available from: www.wellmet2050.com.

Allwood JM, Cullen JM, Cooper DR, Milford RL, Patel ACH, Carruth MA, et al. Conserving our metal energy. University of Cambridge; 2010, ISBN 978-0-903428-30-9, Available from: www.wellmet2050.com.

Ashby MF, Jones DRH. Engineering materials 1: an introduction to properties, applications and design. Cambridge, UK: Elsevier Butterworth-Heinemann; 2005.

Bendsøe MP, Kikuchi N. Generating optimal topologies in structural design using a homogenisation method. Comput Method Appl Mech Eng 1988;71(2):197–224.

Bendsøe MP. Optimal shape design as a material distribution problem. Struct Optimisation 1989;1:193–202.

Bluescope. Bluescope Lysaght product catalogue; 2010, Available from: www.lysaght.com.

Caifu Y. Development of high strength construction rebars. In: Proceedings of international seminar on production and application of high strength seismic grade rebar containing vanadium; 2010. p. 58–70.

Carruth MA. Design optimisation case study: food cans; 2011, Available from: www.wellmet2050.com.

Carruth MA, Allwood JM. The forming of variable cross-section I-beams by hot rolling. In: Proceedings of 10th international conference on technology of plasticity; 2011.

Cheah LW. Cars on a diet: the material and energy impacts of passenger vehicle weight reduction in the U.S. PhD thesis, Massachusetts Institute of Technology; 2010.

Gerst M, Graedel T. In-use stocks of metals: status and implications. Environ Sci Technol 2008;42(19):7038–45.

IEA. Energy technology transitions for industry. Paris: International Energy Agency; 2009, Available from: www.iea.org.

Kim H. New extruded multi-cell aluminum profile for maximum crash energy absorption and weight efficiency. Thin Wall Struct 2002;40(4): 311–27.

Lotus Engineering Inc. An assessment of mass reduction opportunities for a 2017–2020 model year vehicle program. Int Counc Clean Transportation, 2010.

Michell AGM. The limits of economy of material in frame-structures. Philos Mag 6 1904;8(47):589–97.

Morris LJ, Plum DR. Structural steelwork design to BS5950. 2nd ed. Addison Wesley Longman; 1996.

Moynihan M. Design optimisation case study: reinforcing bar; 2011, Available from: www.wellmet2050.com.

Müller D, Wang T, Duval B. Patterns of iron use in societal evolution. Environ Sci Technol 2011;45(1):182–8.

Pedersen P, Pedersen NL. Analytical optimal designs for long and short statically determinate beam structures. Struct Multidiscip Optimization 2009;39: 343–57.

Pulici M, Trifon M, Dumitrescu A. Deep water sealines installation by using the J-lay method – the blue stream experience. In: Proceedings of the 13th international offshore polar engineering conference; 2003. p. 38–43.

Rossow MP, Taylor JE. A finite element method for the optimal design of variable thickness sheets. AIAA J 1973;11(11):1566–9.

Rozvany GI. Grillages of maximum strength and maximum stiffness. Int J Mech Sci 1972;14(10):651–66.

Sigmund O. A 99 line topology optimisation code written in Matlab. Struct Multidiscip Optimization 2001;21(2):120–7.

US Department of Transportation Research and Innovative Technology Administration (USDTRITA). A summary of proceedings for the safety characterisation of future plastic and composite intensive vehicles (PCIVs); 2008.

Vulcraft. Steel joists and joist girders catalogue; 2007, Available from: www.vulcraft.com.

Wanner A. Minimum-weight materials selection for limited available space. Mater Des 2010;31(6):2834–9.

Weaver PM, Ashby MF. Material limits for shape efficiency. Prog Mater Sci 1997;41(1–2):61–128.

World Steel Association. Steel statistical yearbook 2009; 2009.

Xie YM, Steven GP. A simple evolutionary procedure for structural optimisation. Comput Struct 1993;49(5):885–96.